<u>Approximate Methods for Analysis of Indeterminate Structures</u>

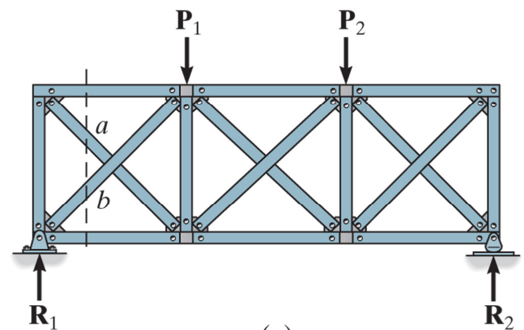
(Ref: Chapter 7)

During preliminary design and analysis, the actual member dimensions are not usually known.

Approximate analysis is useful in determining (approximately) the forces and moments in the different members and in coming up with preliminary designs.
Based on the preliminary design, a more detailed analysis can be conducted and then the design can be refined.
Approximate analysis is conducted by making realistic assumptions about the behavior of the structure.

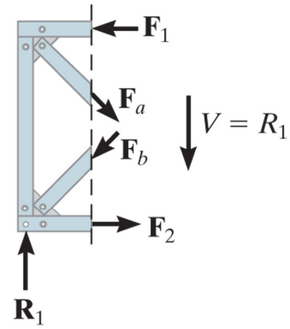
<u>Approximate Analysis of Indeterminate Trusses</u>

<u>Case 1</u>: Diagonals CANNOT carry compression
(because they are designed to be long and slender)
In this case, the truss is usually determinate because only one of the diagonals is active. The "compression" diagonal behaves as a zero-force member.



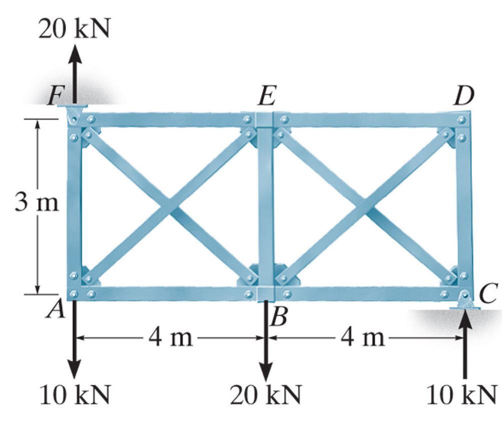
(a)

<u>Case 2</u>: Diagonals CAN carry compression
In this case, both diagonals will be assumed to carry <u>half</u> the panel shear.



(b)

<u>Example</u>
Find the forces in the truss members
- If diagonals <u>cannot</u> carry compression
- If diagonals <u>can</u> carry compression



(a)

Figure: 07_02aEX01

Portal Frames and Trusses

Case 1: Pin supported

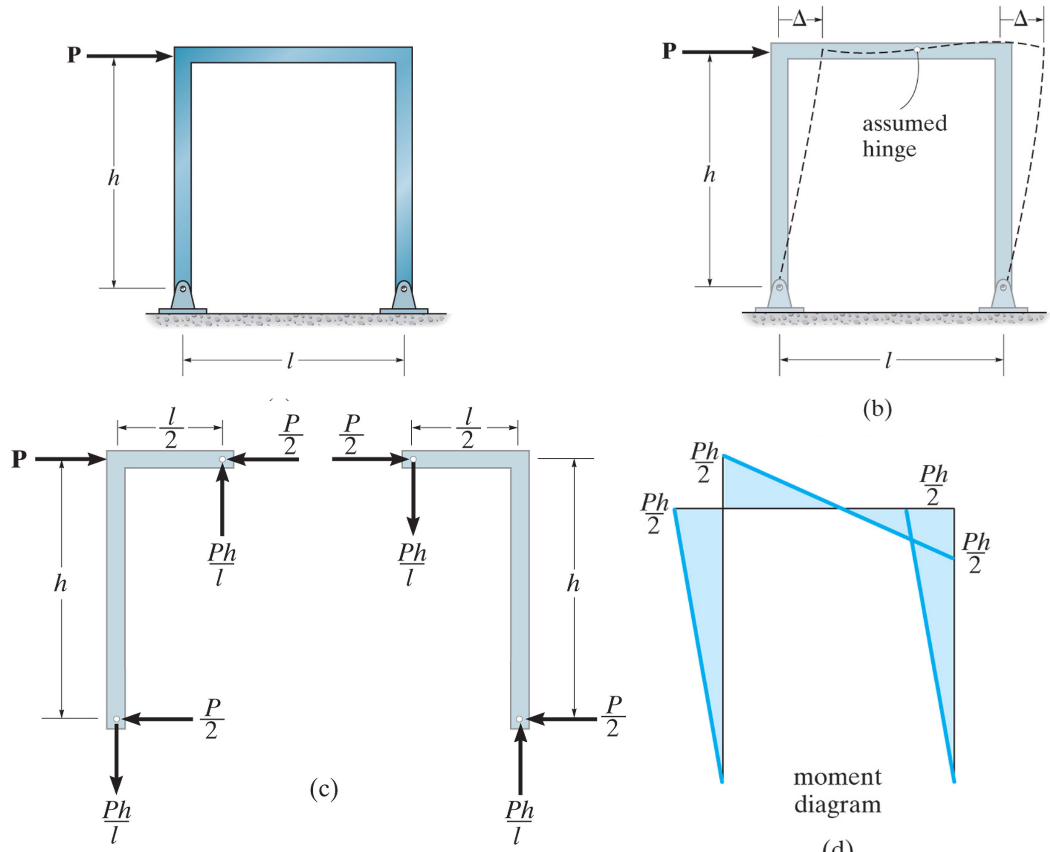


Figure: 07_07c

Case 2: Fixed Supported

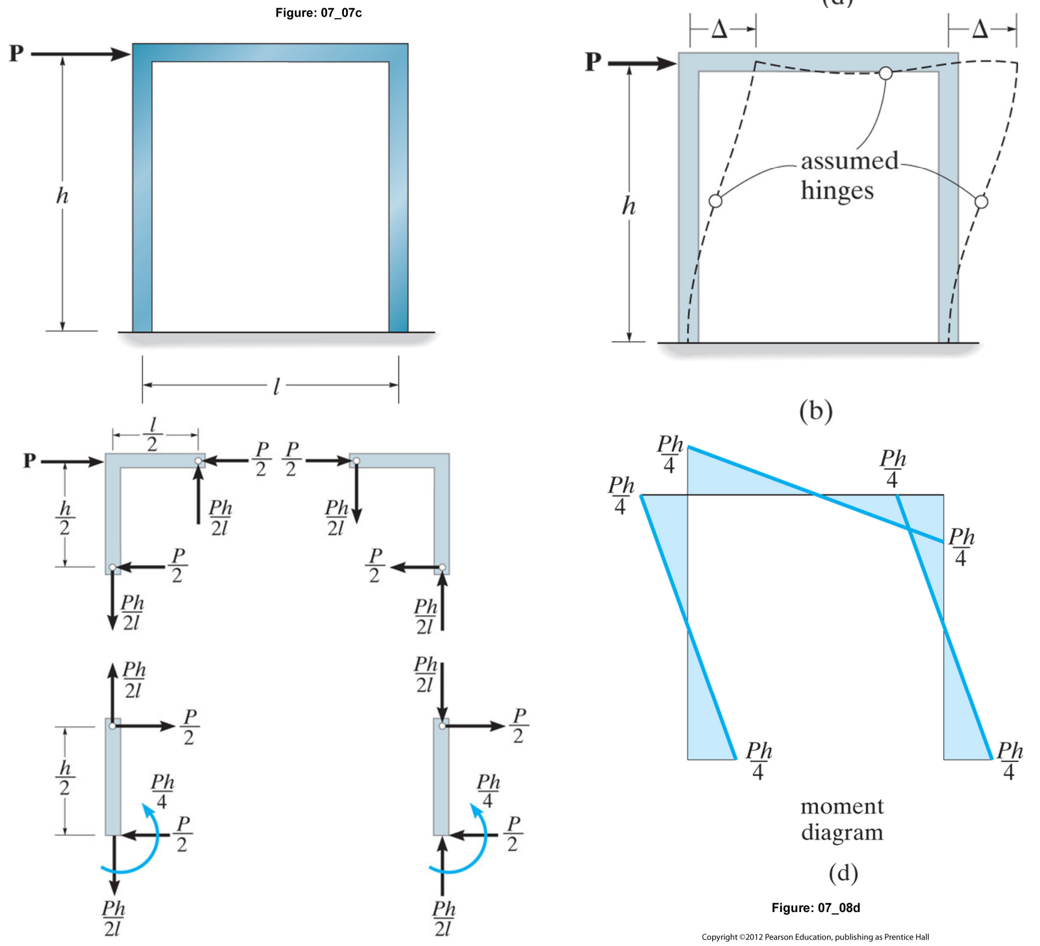
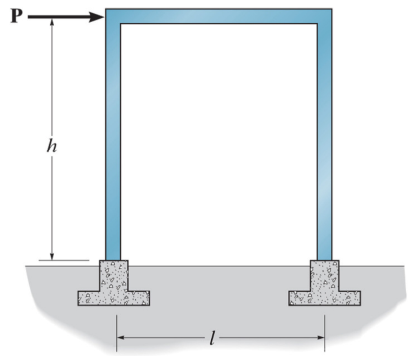


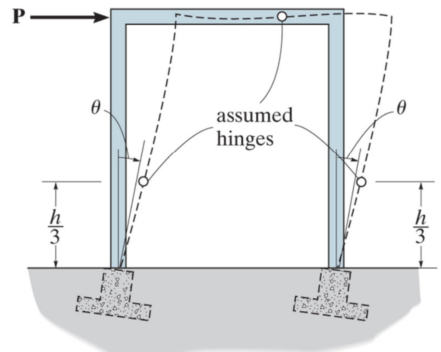
Figure: 07_08d

Case 3: Partial Fixity



(a)

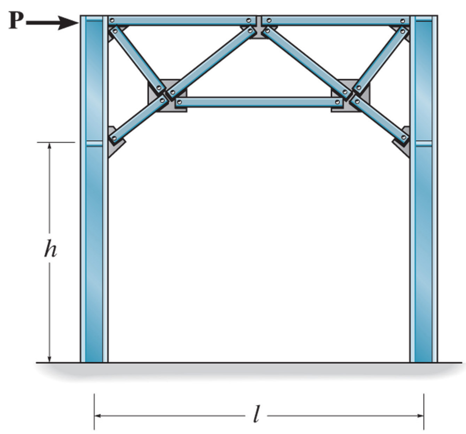
Figure: 07_09a



(b)

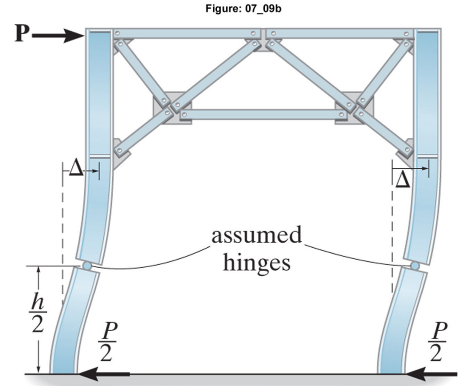
Figure: 07_09b

Case 4: Portal Trusses



(a)

Figure: 07_10a

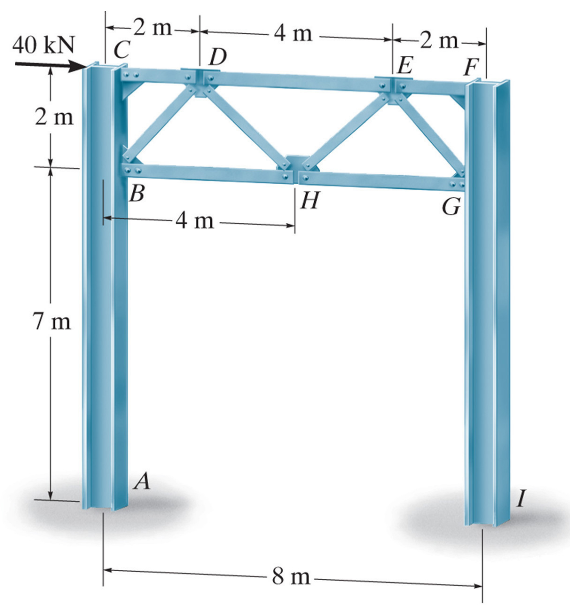


(b)

Figure: 07_10b

Example

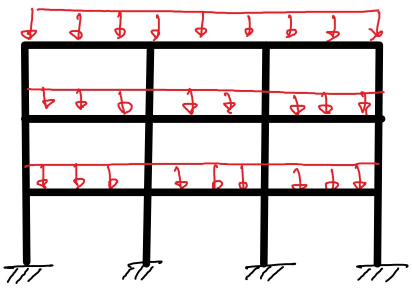
Find the forces in the members of the truss

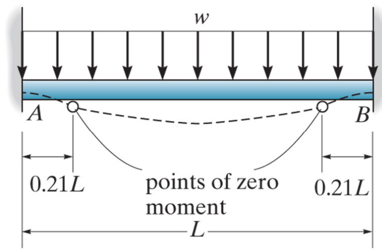


(a)

Figure: 07_11aEX04

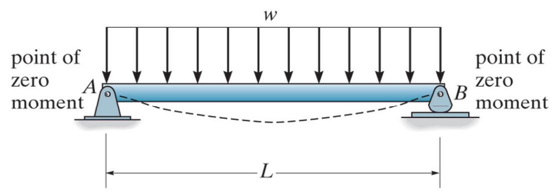
Frame Structures with Vertical Loads





fixed supported
(b)

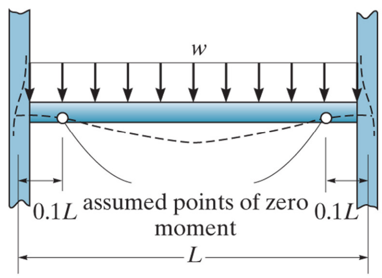
Figure: 07_05b



simply supported
(c)

Figure: 07_05c

Assume hinge at 0.1 L

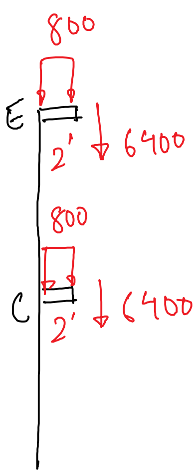


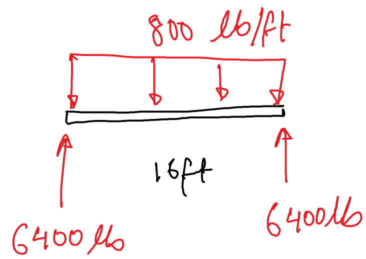
approximate case
(d)

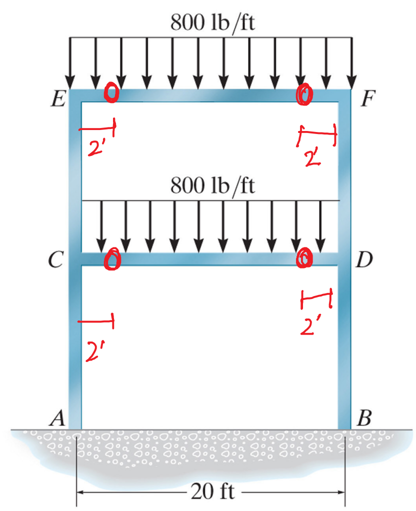
Figure: 07_05d

Example

Find the moments (in the beam) at joints E and C







(a)

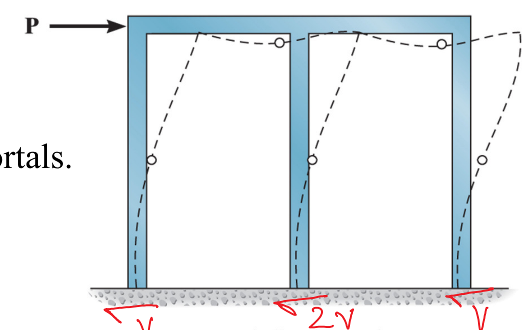
Figure: 07_06aEX03

E:

$$M_E = -6400 \times 2 - (800 \times 2) \times 1$$
$$= -12800 - 1600 = \underline{-14400 \text{ lb-ft}}$$

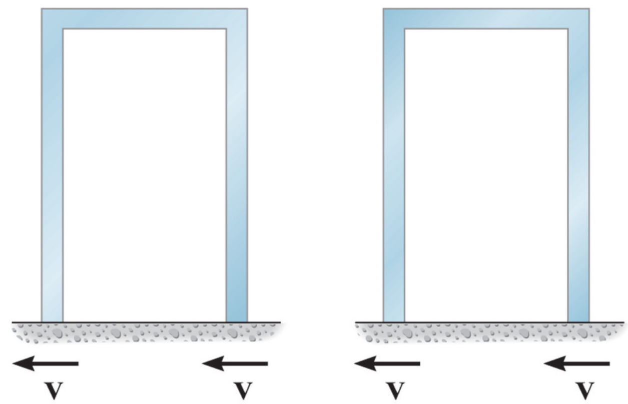
Frame Structures with Lateral Loads: Portal Method

For low-rise building frames under lateral loads,
the frame can be viewed as a superposition of a number of portals.



Assumptions:

- Internal hinges at the centers of beams and columns
- Shear carried by interior columns is assumed to be twice of that of the exterior columns.

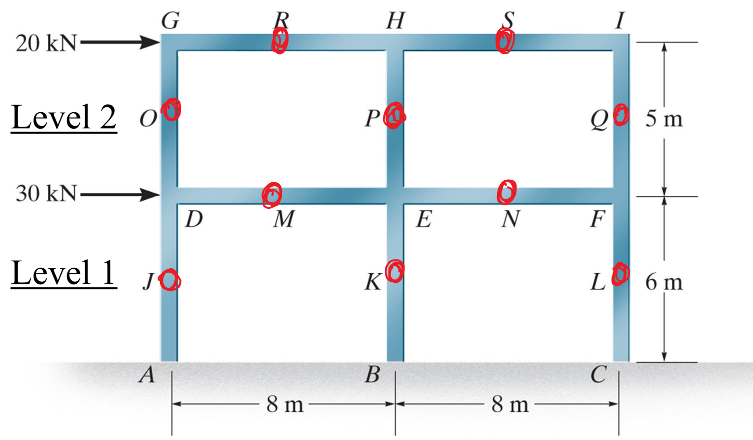


(b)

Figure: 07_12b

Example

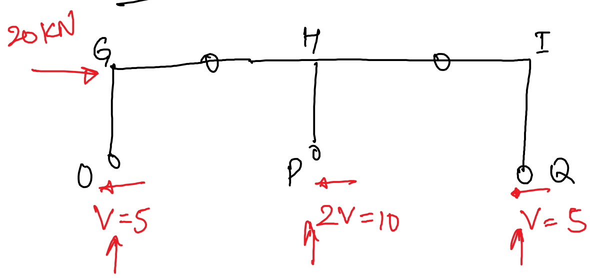
Find the reactions at the base of the columns using the portal method of analysis.

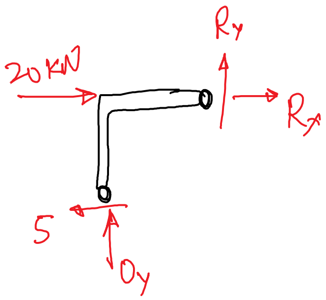


(a)

Figure: 07_14aEX06

2 level



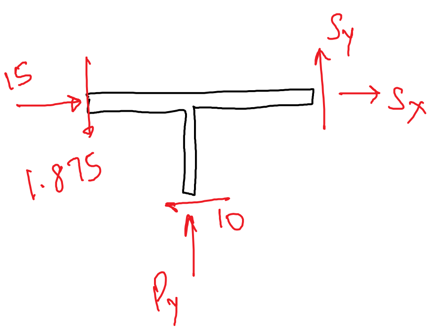


$\sum F_x = 0$

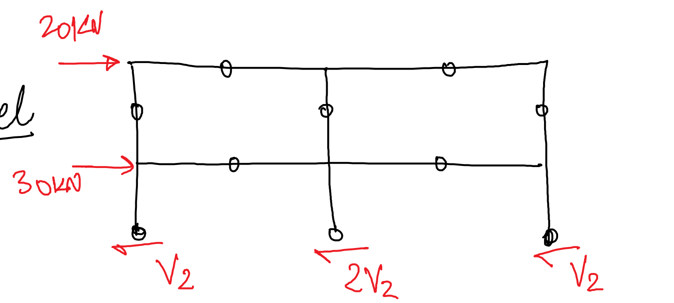
$R_x + 20 - 5 = 0$

$\Rightarrow R_x = -15$ KN

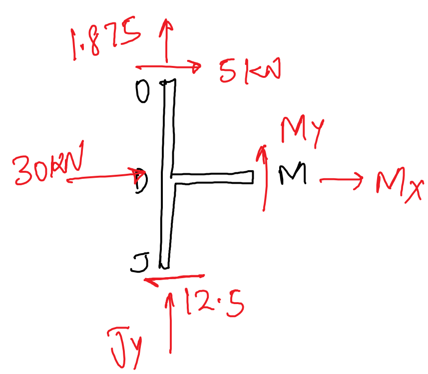
$\sum M_O = -(20 \times 2.5) + (R_y \times 4) + (+15 \times 2.5) \Rightarrow R_y = \frac{\cancel{7.5}}{\cancel{4}}$ 1.875



1-level



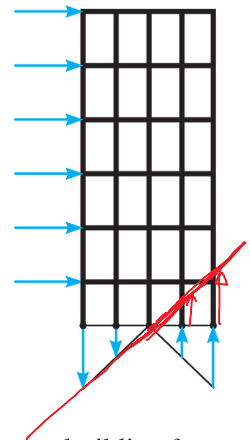
$4V_2 = 50\,KN \Rightarrow V_2 = \frac{50}{4} = 12.5\,KN$



In a similar way, proceed from the top to bottom, analyzing each of the small pieces.

Frame Structures with Lateral Loads: Cantilever Method

For tall and slender building frames under lateral loads,
the entire frame acts similar to cantilever beam sticking out of the ground.
Axial compression and tension forces develop to counteract the moment
created due to the lateral load around the base of the building.



building frame
(b)

Figure: 07_15b

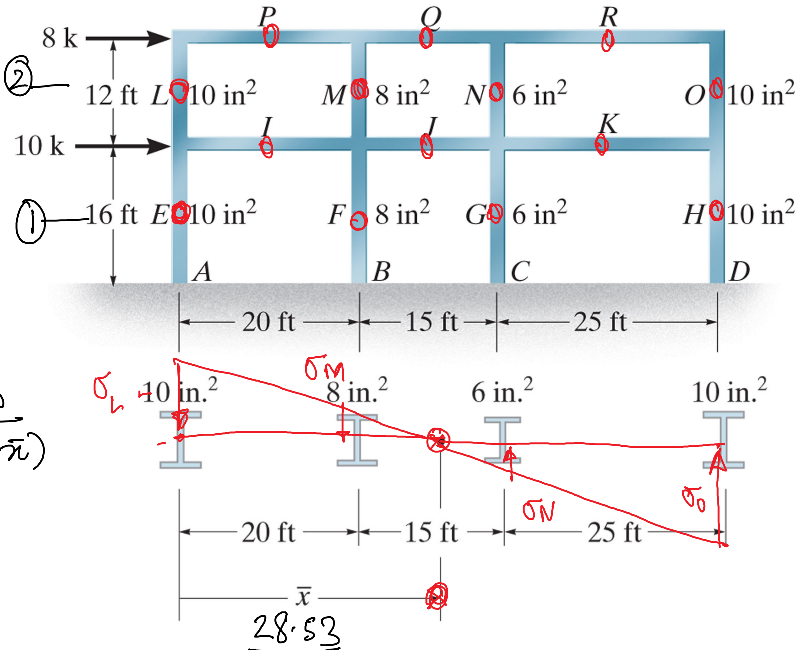
Assumptions:

- Internal hinges form at the center of beams and columns
- Axial $\boxed{\text{stress}}$ in a column is proportional to its distance away from the centroid of the cross-sectional area of columns. at any floor level.

Example

Find the reactions at the base of the columns using the cantilever method of analysis.

Note the areas of cross-sections of the columns are different.

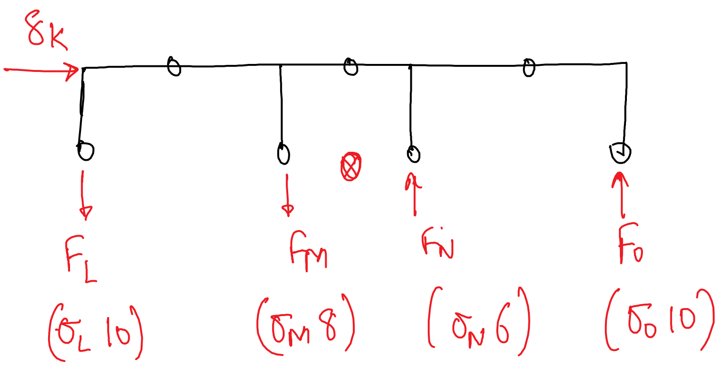


(b)

Figure: 07_17bEX08

2nd level:

$$K = \frac{\sigma_L}{\bar{x}} = \frac{\sigma_M}{(\bar{x}-20)} = \frac{-\sigma_N}{(35-\bar{x})} = \frac{-\sigma_O}{(60-\bar{x})}$$



$$\bar{x} = \frac{(10\times 0)+(8\times 20)+(6\times 35)+(10\times 60)}{(10+8+6+10)}$$

$$= 28.53$$

$$\sum M_{\bar{x}} = 0$$

$$\left.\begin{aligned} &-(8\times 6) \\ &+ (\sigma_L\times 10)\times 28.53 + (\sigma_M\, 8)\times 8.53 \\ &+ (\sigma_N\times 6)\times 6.47 + (\sigma_O\times 10)\times 31.47 = 0 \end{aligned}\right] \Rightarrow K$$

Once κ (the constant of proportionality) is obtained, one can obtain the column forces F_L, F_M, F_N, and F_O.

Then the rest of the analysis would proceed in a similar way to the Portal method, analyzing each of the small pieces between the assumed hinges from top to bottom.

Example

Using the cantilever method, find the reactions at the base of the frame.

All columns have the same cross-sectional area

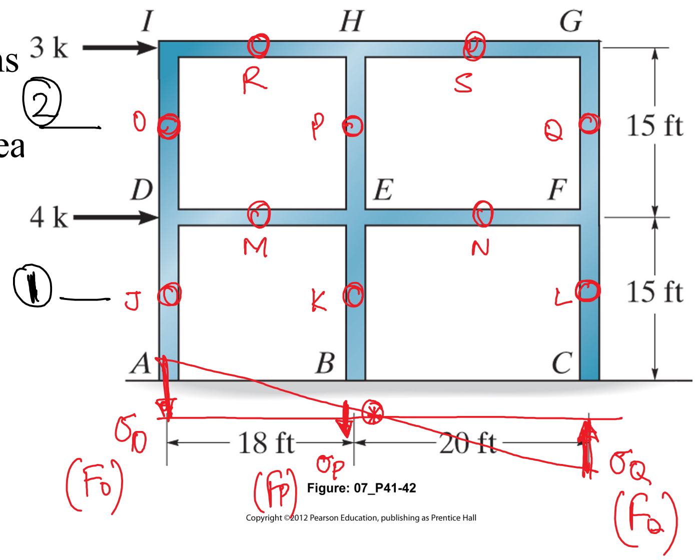
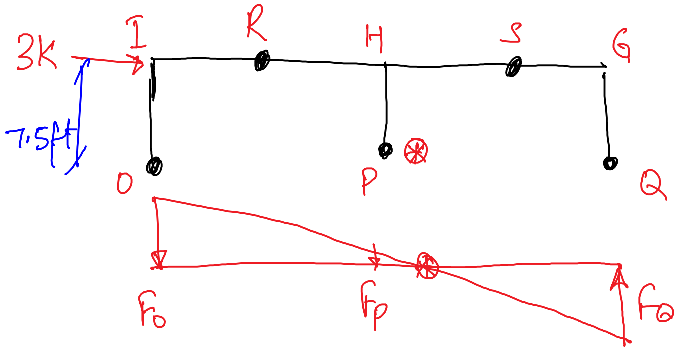


Figure: 07_P41-42

Level (2)

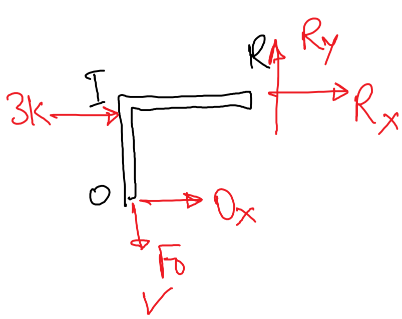


$$\bar{x} = \frac{\cancel{A}\times 0 + \cancel{A}\times 18 + \cancel{A}\times 38}{3\cancel{A}} = \frac{56}{3} = \underline{18.67}$$

$$\sum M_{\bar{x}} = 0 \Rightarrow \left.\begin{aligned} &-3\times 7.5 \\ &+ F_O \times 18.67 + F_P \times 0.67 \\ &+ F_Q \times 19.33 = 0 \end{aligned}\right] \Rightarrow K_2$$

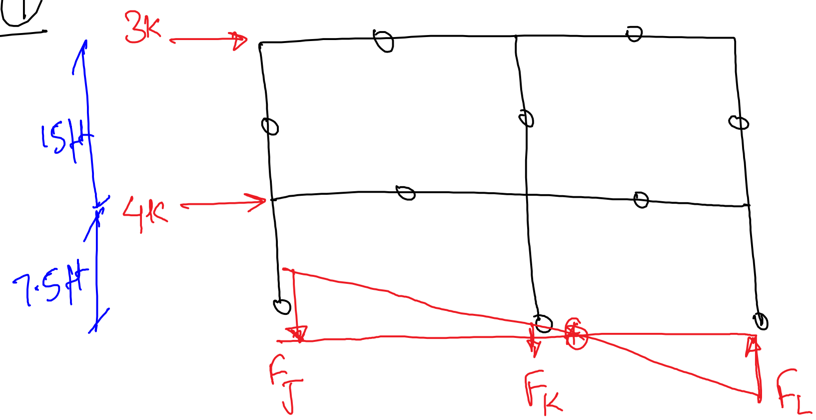
$$\Rightarrow F_O,\ F_P,\ F_Q.$$

$$K_2 = \frac{F_O}{18.67} = \frac{F_P}{0.67} = \frac{F_Q}{19.33}$$



Once again, each of the small pieces will have only 3 unknowns and can be solved for using Statics.

Level (1)



$$\sum M_{\bar{x}} = 0 \Rightarrow -3 \times 22.5 - 4 \times 7.5 + F_J \times 18.67 + F_K \times 0.67 + F_L \times 19.33 \Rightarrow K_1 \Rightarrow F_J, F_K, F_L$$

$$K_1 = \frac{F_J}{18.67} = \frac{F_K}{0.67} = \frac{F_L}{19.33}$$

In a similar way, proceed from the top to bottom, analyzing each of the small pieces.