CE 570: Advanced Structural Mechanics

HOMEWORK 3

Part 1: Due ONLINE on blackboard on at 11:30am Saturday, Sep 23, 2016
Part 2: Due ONLINE on blackboard **and** in class at 11:30am Monday, Sep 25, 2016

Part 1 guidelines:

- Work your solution **independently** and **neatly,** on **one side** only on college-rule / engineering paper.
- You may use any combination of mix of **black / blue / green** pens or pencils (but not red).
- Start every problem on a **new** page.
- All **diagrams** must be drawn **neatly** using a straight edge.
- All work should be presented in a **logical sequence**.
- **Scan & submit your homework online** on Blackboard as a **single pdf-file**.
- **Do not email** your homework to the instructor.
- Make sure that your **scan is good quality** and your pdf-file is **clearly readable.**
  Cell-phone / camera pictures of your homework will **not** be accepted / graded.
  Illegible or light scans will **not** be graded**.**
- All the scans must be in a **single pdf-file**. To edit, combine or create pdf-files you may use any of the following freely software programs:
  - *PDF Architect* and/or *PDF Creator* (http://www.pdfforge.org/)
  - *Primo-pdf* (http://www.primopdf.com)

  Try to make sure that your pdf-file size is not more than 5MB (Maximum 10MB).
- The **file name** of your scan must be in the format "`HW??-FirstLast-1.pdf`" where "`??`" is the HW number, "`First`" and "`Last`" are your first and last names, and the "`-1`" denotes Part 1. e.g. `HW01-ArunPrakash-1.pdf`.

Part 2 guidelines: (Work in red pen only)

- The solutions will be posted online at 5pm on Friday (on the due date for Part-1).
- Based on the posted solutions:
  - Correct any errors in your work and revise your solution. If you made any errors, comment why you think you made the error(s) and how you will avoid such error(s) in the future.
  - For each problem, list the most important concepts that you learned.
  - Briefly comment how you may be able to verify / cross-check your revised solution and the posted solution. Also comment, if you think that the posted solution is incorrect.
- You may add pages if necessary, but do **not** submit an entirely new homework file for Part 2.
- **Scan & submit your revised homework online** on Blackboard as a **single pdf-file**.
- The **file name** of your scan must be in the format "`HW??-FirstLast-2.pdf`"

Grading & Solutions:

- **Part 1**: 10 points = 3 problems x 3 points each + 1 presentation point
  - For Part-1, we will grade based only on your effort: You can get full 3 points for a problem, if you made an **honest independent effort** (even if your solution was incorrect!).
- **Part 2**: 5 points (for revisions and comments)
- Total: **15 points**

CE 570: Advanced Structural Mechanics

HW Guidelines:

- Review multi-variate calculus: line, area and volume integrals, partial derivatives etc.
- Read Chapter 1 (again!) from *Fundamentals of Structural Mechanics* by KD Hjelmstad.
- Work your solution neatly, starting all the problems on a new page.
- Be **very precise** with notation. You will lose ½ point for every notational error that you make. So, if you make 10 notational errors in 1 question, you will receive a *zero* score even though your solution may have the right idea.

Problem 1: (5 points)
Using indicial notation (and expanding if necessary), solve Problem 31 from the textbook:

31. Evaluate the following expressions:

(a) $\operatorname{div}\big(\operatorname{div}[\mathbf{x}\otimes\mathbf{x}]\big)$ (b) $\operatorname{div}\big(\mathbf{x}\operatorname{div}(\mathbf{x}\operatorname{div}\mathbf{x})\big)$ (c) $\nabla\big[\|\,(\nabla\|\mathbf{x}\|^2)\,\|^2\big]$

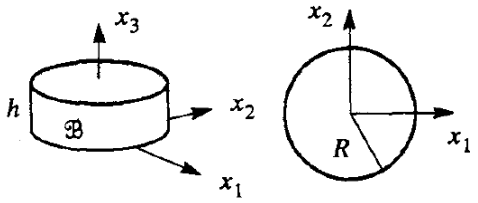
(d) $\operatorname{div}\big(\mathbf{x}\otimes\operatorname{div}[\mathbf{x}\otimes\mathbf{x}]\big)$ (e) $\nabla(\mathbf{x}\operatorname{div}\mathbf{x})$ (f) $\nabla\big[\mathbf{x}\cdot\nabla(\mathbf{x}\cdot\mathbf{x})\big]$

where $\mathbf{x} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2 + x_3\mathbf{e}_3$ is the position vector in space and all derivatives are with respect to the coordinates x_i.

Problem 2: (5 points)
Solve Problem 34 from the textbook:

34. Let $\mathbf{v}(\mathbf{x})$ be a vector field given by the following explicit function

$$\mathbf{v}(\mathbf{x}) = (x_1\mathbf{e}_1 + x_2\mathbf{e}_2)\ln(x_1^2 + x_2^2)$$



where $\ln(\cdot)$ indicates the natural logarithm of $(\cdot)$. The vector field is defined on the cylindrical region $\mathfrak{B}$ of height h and radius R as shown in the sketch. Give an expression for the unit normal vector field $\mathbf{n}(\mathbf{x})$ to the for the cylinder (including the ends). Compute the divergence of the vector field $\mathbf{v}(\mathbf{x})$ and the integral of div $\mathbf{v}$ over the volume of the cylinder.

Problem 3: (5 points)
Solve Problem 39 from the textbook:

39. Consider a tensor field $\mathbf{T}$ defined on a tetrahedral region bounded by the coordinate planes $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, and the oblique plane $6x_1 + 3x_2 + 2x_3 = 6$, as shown in the sketch. The tensor field has the particular expression $\mathbf{T} = \mathbf{b}\otimes\mathbf{x}$, where $\mathbf{b}$ is a constant vector and $\mathbf{x}$ is the position vector $\mathbf{x} = x_i\,\mathbf{e}_i$. Compute the integral of $\operatorname{div}(\mathbf{T})$ over the volume and the integral of $\mathbf{Tn}$ over the surface of the tetrahedron (and thereby show that they give the same result, as promised by the divergence theorem). Note that the volume of the tetrahedron of the given dimensions is one.