CE 570: Advanced Structural Mechanics

HOMEWORK 5

Part 1: Due ONLINE on blackboard on at 11:30am Saturday, Oct 14, 2016
Part 2: Due ONLINE on blackboard **and** in class at 11:30am Monday, Oct 16, 2016

Part 1 guidelines:

- Work your solution **independently** and **neatly,** on **one side** only on college-rule / engineering paper.
- You may use any combination of mix of **black / blue / green** pens or pencils (but not red).
- Start every problem on a **new** page.
- All **diagrams** must be drawn **neatly** using a straight edge.
- All work should be presented in a **logical sequence**.
- **Scan & submit your homework online** on Blackboard as a **single pdf-file**.
- **Do not email** your homework to the instructor.
- Make sure that your **scan is good quality** and your pdf-file is **clearly readable.** Cell-phone / camera pictures of your homework will **not** be accepted / graded. Illegible or light scans will **not** be graded**.**
- All the scans must be in a **single pdf-file**. To edit, combine or create pdf-files you may use any of the following freely software programs:
    - *PDF Architect* and/or *PDF Creator* (http://www.pdfforge.org/)
    - *Primo-pdf* (http://www.primopdf.com)

  Try to make sure that your pdf-file size is not more than 5MB (Maximum 10MB).
- The **file name** of your scan must be in the format "`HW??-FirstLast-1.pdf`" where "`??`" is the HW number, "`First`" and "`Last`" are your first and last names, and the "`-1`" denotes Part 1. e.g. `HW01-ArunPrakash-1.pdf`.

Part 2 guidelines: (Work in red pen only)

- The solutions will be posted online at 5pm on Friday (on the due date for Part-1).
- Based on the posted solutions:
    - Correct any errors in your work and revise your solution. If you made any errors, comment why you think you made the error(s) and how you will avoid such error(s) in the future.
    - For each problem, list the most important concepts that you learned.
    - Briefly comment how you may be able to verify / cross-check your revised solution and the posted solution. Also comment, if you think that the posted solution is incorrect.
- You may add pages if necessary, but do **not** submit an entirely new homework file for Part 2.
- **Scan & submit your revised homework online** on Blackboard as a **single pdf-file**.
- The **file name** of your scan must be in the format "`HW??-FirstLast-2.pdf`"

Grading & Solutions:

- **Part 1**: 10 points = 3 problems x 3 points each + 1 presentation point
    - For Part-1, we will grade based only on your effort: You can get full 3 points for a problem, if you made an **honest independent effort** (even if your solution was incorrect!).
- **Part 2**: 5 points (for revisions and comments)
- Total: **15 points**

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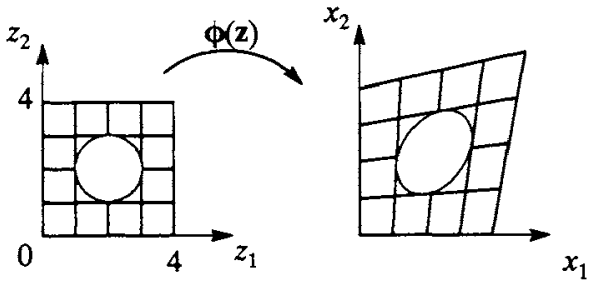
HW Guidelines:

- Read Chapter 2 (again!) from *Fundamentals of Structural Mechanics* by KD Hjelmstad.
- Work your solution neatly, starting all the problems on a new page.
- Be **very precise** with notation. You will lose ½ point for every notational error that you make. So, if you make 10 notational errors in 1 question, you will receive a *zero* score even though your solution may have the right idea.

Problem 1: (5 points)

Solve Problem 51 from the textbook:

51. Consider a square piece of material of unit thickness with a round hole in it of radius 1. The material is subjected to a deformation described by the map shown in the diagram. The deformation map shown has the following explicit expression



$$\phi(\mathbf{z}) = z_1(1+\beta z_2)\mathbf{e}_1 + z_2(1+3\beta z_1)\mathbf{e}_2 + z_3\mathbf{e}_3$$

Compute the the volume of the hole in the undeformed and deformed configurations.
Compute the perimeter area of the square in the undeformed and deformed configurations.
Compute the perimeter area of the circle in the undeformed and deformed configurations.

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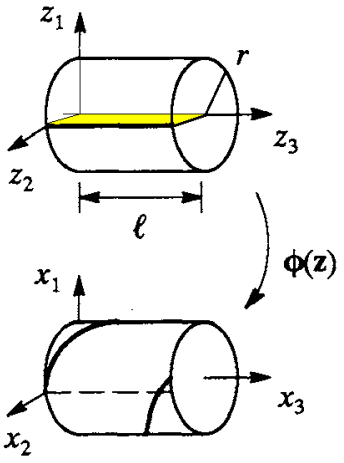
Problem 2: (5 points)

a) Solve Problem 61 from the textbook:

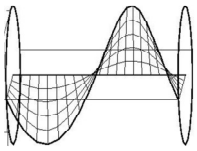
61. The deformation map for the pure twist of a circular shaft of length ℓ and radius r can be expressed in terms of the rate of twist β (a constant) as follows

$$\boldsymbol{\phi}(\mathbf{z}) = \left(z_1 \cos(\beta z_3) - z_2 \sin(\beta z_3)\right) \mathbf{e}_1 + \left(z_1 \sin(\beta z_3) + z_2 \cos(\beta z_3)\right) \mathbf{e}_2 + z_3 \mathbf{e}_3$$

Compute the deformation gradient $\mathbf{F}(\mathbf{z})$. Find the displacement of the point initially located at the position $\mathbf{z} = (r, 0, \ell)$ in the undeformed configuration. Find the volume of the deformed shaft in terms of the angle of twist β. A horizontal line is etched on the surface of the undeformed shaft, parallel to the z_3 axis as shown. Find the length of the line in the deformed configuration.



b) Write an integral expression for the deformed area for the original area shaded as shown.



c) Compare the expression of the deformed area to the area of a spiraling helical surface characterized by the parameterization:

$$S : (s,\theta) \mapsto \begin{Bmatrix} x_1(s,\theta) = s\cos(\theta) \\ x_2(s,\theta) = s\sin(\theta) \\ x_3(s,\theta) = \frac{\theta}{2\pi} l \end{Bmatrix} \text{ where } \begin{matrix} 0 \le s \le r \\ 0 \le \theta \le 2\pi \end{matrix}$$

when the rate of twist $\beta = 2\pi/l$.

Problem 3: (5 points)

a) Read the documentation for functions `integral()`, `integral2()`, `integral3()`, `quad()`, `dblquad()`, `triplequad()` in MATLAB Help

b) Use these functions to numerically evaluate all the integrals for the deformed configurations in Problem 1 (volume of hole, perimeter area of the square, perimeter area of the circle). Assume $\beta = 0.2$.

c) Use these functions to numerically evaluate all the integrals for the deformed configurations in Problem 2 (volume of cylindrical shaft, length of the etched line, area of the surface). Assume $l = 5$; $r = 1$.

d) Submit a printout of your MATLAB code and the output it generates.