CE 570: Advanced Structural Mechanics

HOMEWORK 6

Part 1: Due ONLINE on blackboard on at 11:30am Saturday, Oct 21, 2016
Part 2: Due ONLINE on blackboard **and** in class at 11:30am Monday, Oct 23, 2016

Part 1 guidelines:

- Work your solution **independently** and **neatly,** on **one side** only on college-rule / engineering paper.
- You may use any combination of mix of **black / blue / green** pens or pencils (but not red).
- Start every problem on a **new** page.
- All **diagrams** must be drawn **neatly** using a straight edge.
- All work should be presented in a **logical sequence**.
- **Scan & submit your homework online** on Blackboard as a **single pdf-file**.
- **Do not email** your homework to the instructor.
- Make sure that your **scan is good quality** and your pdf-file is **clearly readable.**
  Cell-phone / camera pictures of your homework will **not** be accepted / graded.
  Illegible or light scans will **not** be graded**.**
- All the scans must be in a **single pdf-file**. To edit, combine or create pdf-files you may use any of the following freely software programs:
  - *PDF Architect* and/or *PDF Creator* (http://www.pdfforge.org/)
  - *Primo-pdf* (http://www.primopdf.com)

  Try to make sure that your pdf-file size is not more than 5MB (Maximum 10MB).
- The **file name** of your scan must be in the format "`HW??-FirstLast-1.pdf`" where "`??`" is the HW number, "`First`" and "`Last`" are your first and last names, and the "`-1`" denotes Part 1. e.g. `HW01-ArunPrakash-1.pdf`.

Part 2 guidelines: (Work in red pen only)

- The solutions will be posted online at 5pm on Friday (on the due date for Part-1).
- Based on the posted solutions:
  - Correct any errors in your work and revise your solution. If you made any errors, comment why you think you made the error(s) and how you will avoid such error(s) in the future.
  - For each problem, list the most important concepts that you learned.
  - Briefly comment how you may be able to verify / cross-check your revised solution and the posted solution. Also comment, if you think that the posted solution is incorrect.
- You may add pages if necessary, but do **not** submit an entirely new homework file for Part 2.
- **Scan & submit your revised homework online** on Blackboard as a **single pdf-file**.
- The **file name** of your scan must be in the format "`HW??-FirstLast-2.pdf`"

Grading & Solutions:

- **Part 1**: 10 points = 3 problems x 3 points each + 1 presentation point
  - For Part-1, we will grade based only on your effort: You can get full 3 points for a problem, if you made an **honest independent effort** (even if your solution was incorrect!).
- **Part 2**: 5 points (for revisions and comments)
- Total: **15 points**

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HW Guidelines:

- Read Chapters 2 and 3 from *Fundamentals of Structural Mechanics* by KD Hjelmstad.
- Work your solution neatly, starting all the problems on a new page.
- Be **very precise** with notation. You will lose ½ point for every notational error that you make. So, if you make 10 notational errors in 1 question, you will receive a *zero* score even though your solution may have the right idea.

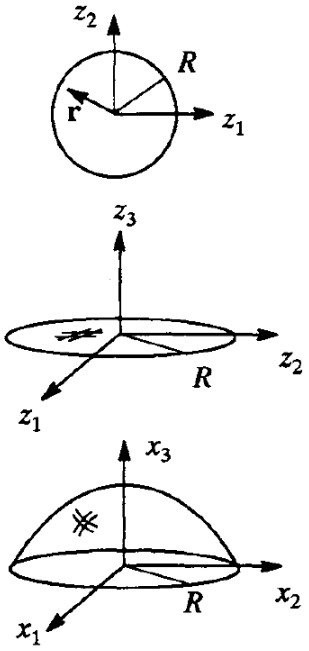
Problem 1: (5 points)

Solve Problem 79 from the textbook:

79. Consider a thin (i.e., it has essentially no thickness in the z_3 direction) circular membrane of radius R initially lying in the z_1-z_2 plane as shown in the sketch. Under pressure the membrane deforms into a bubble according to the following deformation map

$$\phi(\mathbf{z}) = z_1\mathbf{e}_1 + z_2\mathbf{e}_2 + \beta\cos\left(\pi\sqrt{z_1^2+z_2^2}/2R\right)\mathbf{e}_3$$

where β is a known constant and R is the radius of the circle. Compute the deformation gradient of the given map. Compute the stretch in the initial radial direction (i.e., the direction of the vector $\mathbf{r} = z_1\mathbf{e}_1 + z_2\mathbf{e}_2$). Also compute the stretch in the direction that is in the initial plane of the membrane but is orthogonal to $\mathbf{r}$ (i.e., tangent to a circle centered at the origin). Are these two directions principal directions? Why or why not? What is the deformed length of the line that was the radial line from the origin to the edge of the circle along the z_1 direction in the undeformed configuration? What is the slope of the membrane at the edge after deformation?



Note: The stretch through the thickness of the membrane is zero, but that is acceptable because we are assuming that the thickness is very small compared to the diameter of the membrane.

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Problem 2: (5 points)
Solve Problem 81 from the textbook:

81. The stress tensor **S** at a certain point in a body has components with respect to a set of coordinate axes $\{x_1, x_2, x_3\}$ of

$$\mathbf{S} \sim \begin{bmatrix} 5 & 3 & -8 \\ 3 & 0 & -3 \\ -8 & -3 & 11 \end{bmatrix}$$

On a plane whose normal **n** makes equal acute angles with the coordinate axes, find the traction vector $\mathbf{t_n}$, the component of the traction vector that is normal to the plane, and the shearing component of the traction vector.

Problem 3: (5 points)
Solve Problem 85 from the textbook:

85. The state of stress at a point is characterized by the stress tensor **S**, given below

$$\mathbf{S} \sim \begin{bmatrix} 4 & -4 & 0 \\ -4 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

Consider the vectors **n** and **m** given by

$$\mathbf{n} = \frac{1}{\sqrt{3}}(\mathbf{e}_1 - \mathbf{e}_2 - \mathbf{e}_3), \quad \mathbf{m} = \frac{1}{\sqrt{2}}(\mathbf{e}_1 + \mathbf{e}_2)$$

Are the two given vectors **n** and **m** eigenvectors of **S**? Find the principal stresses for the given stress tensor **S**.