CE 570: Advanced Structural Mechanics

HOMEWORK 9

Part 1: Due ONLINE on blackboard on at 11:30am Wednesday, Nov 29, 2017

Part 2: Due ONLINE on blackboard **AND** IN CLASS at 11:30am Friday, Dec 1, 2017

Part 1 guidelines:

- Work your solution **independently** and **neatly,** on **one side** only on college-rule / engineering paper.
- You may use any combination of mix of **black / blue / green** pens or pencils (but not red).
- Start every problem on a **new** page.
- All **diagrams** must be drawn **neatly** using a straight edge.
- All work should be presented in a **logical sequence**.
- **Scan & submit your homework online** on Blackboard as a **single pdf-file**.
- **Do not email** your homework to the instructor.
- Make sure that your **scan is good quality** and your pdf-file is **clearly readable.** Cell-phone / camera pictures of your homework will **not** be accepted / graded. Illegible or light scans will **not** be graded**.**
- All the scans must be in a **single pdf-file**. To edit, combine or create pdf-files you may use any of the following freely software programs:
  - *PDF Architect* and/or *PDF Creator* (http://www.pdfforge.org/)
  - *Primo-pdf* (http://www.primopdf.com)

  Try to make sure that your pdf-file size is not more than 5MB (Maximum 10MB).
- The **file name** of your scan must be in the format "`HW??-FirstLast-1.pdf`" where "`??`" is the HW number, "`First`" and "`Last`" are your first and last names, and the "`-1`" denotes Part 1. e.g. `HW01-ArunPrakash-1.pdf`.

Part 2 guidelines: (Work in red pen only)

- The solutions will be posted online at 5pm on Friday (on the due date for Part-1).
- Based on the posted solutions:
  - Correct any errors in your work and revise your solution. If you made any errors, comment why you think you made the error(s) and how you will avoid such error(s) in the future.
  - For each problem, list the most important concepts that you learned.
  - Briefly comment how you may be able to verify / cross-check your revised solution and the posted solution. Also comment, if you think that the posted solution is incorrect.
- You may add pages if necessary, but do **not** submit an entirely new homework file for Part 2.
- **Scan & submit your revised homework online** on Blackboard as a **single pdf-file and also submit the hard paper copy in class** on the following Monday.
- The **file name** of your scan must be in the format "`HW??-FirstLast-2.pdf`"

Grading & Solutions:

- **Part 1**: 9 points = 3 problems x 3 points each
  - For Part-1, we will grade based only on your effort: You can get full 3 points for a problem, if you made an **honest independent effort** (even if your solution was incorrect!).
- **Part 2**: 6 points = 3 problems x 2 points each (for revisions and comments)
- Total: **15 points**

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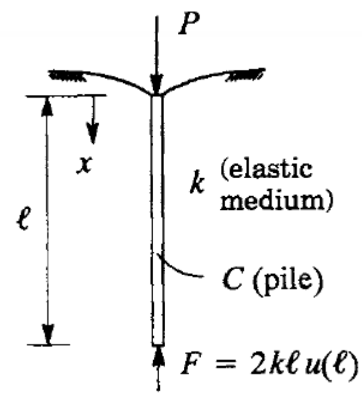
HW Guidelines:

- Read Chapters 5, 6 and 9 from *Fundamentals of Structural Mechanics* by KD Hjelmstad.
- Work your solution neatly, starting all the problems on a new page.
- Be **very precise** with notation. You will lose ½ point for every notational error that you make. So, if you make 10 notational errors in 1 question, you will receive a *zero* score even though your solution may have the right idea.

Problem 1: (5 points)

Solve Problem 137 from the textbook:

137. Consider the pile of length ℓ, constant modulus C (w/ unit area), embedded in an elastic medium with modulus k (force per unit displacement per unit length), and subjected to a load P at $x = 0$. The pile is elastically restrained at the end $x = \ell$ giving an end force of the amount $F = 2k\ell u(\ell)$ as shown. The governing differential equation for the system is $Cu''(x) - ku(x) = 0$. What must be the value of the constant a for the solution to have the form $u(x) = Ae^{ax} + Be^{-ax}$?



What are the values of the constants A and B that satisfy the problem shown in the figure? Does this function $u(x)$ represent a classical solution to the given problem? Why or why not? Are there any other solutions to this specific problem?

a) First write the boundary conditions at $x = 0$ and $x = l$.
Note: The boundary condition at $x = l$ is like having a spring with stiffness $(2kl)$ at the end.

b) Then solve the different parts of the problem stated above.

c) Use MATLAB to plot the exact solution for displacement $u(x)$ and the stress $\sigma(x)$ using the following values of the problem parameters: $l = 10$, $C = 100$, $k = 5$, $P = 20$ units.
Submit printouts of your MATLAB code and output as well.

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Problem 2: (5 points)
For Problem 1,

a) Derive the Method of Weighted Residuals (MWR) weak form and the Principle of Virtual Work (PVW) weak form of the problem statement. Define the functional $\tilde{G}(u,\overline{u})$ and clearly specify the function spaces for $u(x)$ and $\overline{u}(x)$, and the type of boundary conditions in each case.

b) For the PVW weak form, if we restrict the space of functions for $u(x)$ and $\overline{u}(x)$ to polynomials

i.e.
$$u(x) \cong \hat{u}(x) = a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + a_n x^n$$
$$\overline{u}(x) \cong \hat{\overline{u}}(x) = \overline{a}_0 x^0 + \overline{a}_1 x^1 + \overline{a}_2 x^2 + \cdots + \overline{a}_n x^n$$

Comment if this approximation belongs to the function spaces in part (a) above.
Is it possible to obtain the exact solution in Problem 1 using this approximation for any finite value of n?

c) Find an approximate solution to this problem using the above approximations (with $n = 0$),

i.e. assume $u(x) \cong a_0$ and $\overline{u}(x) \cong \overline{a}_0$

Integrate the PVW weak form to get an algebraic equation of the form:
$$\overline{a}_0[K_{00}\, a_0 - f_0] = 0 \quad \forall \overline{a}_0$$
where K_{00} and f_0 can be found in terms of l, C, k, and P (values given in Problem 1(c)).
Finally, solve for a_0 from the above equation.

d) Find another approximate solution to this problem using the same approximation (with $n = 1$),

i.e. assume $u(x) \cong a_0 + a_1 x$ and $\overline{u}(x) \cong \overline{a}_0 + \overline{a}_1 x$

Integrate the PVW weak form to get two algebraic equations of the form:
$$\left\{\begin{matrix} \overline{a}_0[K_{00}\, a_0 + K_{01}\, a_1 - f_0] = 0 & \forall \overline{a}_0 \\ \overline{a}_1[K_{10}\, a_0 + K_{11}\, a_1 - f_1] = 0 & \forall \overline{a}_1 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \overline{a}_0 & \overline{a}_1 \end{bmatrix} \left\{ \begin{bmatrix} K_{00} & K_{01} \\ K_{10} & K_{11} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} - \begin{bmatrix} f_0 \\ f_1 \end{bmatrix} \right\} = 0 \quad \forall \overline{a}_0, \overline{a}_1$$
where again K_{00}, K_{01}, K_{10}, K_{11}, f_0, and f_1, can be found in terms of l, C, k, and P.
Finally, solve for a_0 and a_1 from the two equations above.

e) Plot the two approximate solutions above on the same plot as part (c) of problem 1 and comment on your results.

Problem 3: (5 points)
For the strong form in Problem 1 and the corresponding weak form in Problem 2 above,

a) Show that an energy functional $\Pi(u)$ exists, and find it.

b) Show that by minimizing this energy functional $\Pi(u)$ one obtains the weak form in Problem 2.

c) Find the Euler-Lagrange equation and boundary conditions for this energy functional $\Pi(u)$.