CE 506 Homework 4 Solution – 27 Sept. 2003

Problem 1(a) angle figure, observations only, equal precision

$n = 8$
$n_0 = 4$
$r = 4$

$\hat{\ell}_1 + \hat{\ell}_2 + \hat{\ell}_3 = 360^\circ$
$\hat{\ell}_2 + \hat{\ell}_8 = \hat{\ell}_7$
$\hat{\ell}_6 + \hat{\ell}_5 = \hat{\ell}_1$
$\hat{\ell}_4 = 80^\circ + \hat{\ell}_5$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix}, \quad d = \begin{bmatrix} 360 \\ 0 \\ 0 \\ 80 \end{bmatrix}, \quad W = I_8$$

Matlab code

```
% hw41a.m  ce506 - '03
% 27-sep-03

% set up the problem
% solve by observations only

n=8;
n0=4;
r=4;
c=r;

l=[73+35/60; 130+55/60; 155+40/60; 102+54/60;
   23+15/60; 50+20/60; 146+25/60; 15+40/60];
A=[1 1 1 0 0 0 0 0;
   0 1 0 0 0 0 -1 1;
   -1 0 0 0 1 1 0 0;
   0 0 0 1 -1 0 0 0];
d=[360; 0; 0; 80];
W=eye(n);
f=d - A*l;

% now solve it

Mtx=[W -A'; -A zeros(c,c)];
Vec=[zeros(n,1); -f];
Sol=inv(Mtx)*Vec;
v=Sol(1:n);
k=Sol(n+1:n+c);
lhat=l+v;

l_deg=fix(l);
l_min=(l-l_deg)*60;
lhat_deg=fix(lhat);
lhat_min=(lhat-lhat_deg)*60;
v_min=v*60;
chk=A*v - f;

disp('original observations, decimal degrees');
l
disp('original observations, d-m');
[l_deg l_min]
disp('weight matrix');
W
```

```
disp('A matrix');
A
disp('f vector');
f

disp('residuals, decimal degrees');
v
disp('residuals, d-m');
v_min
disp('lagrange multipliers');
k
disp('adjusted observations, decimal degrees');
lhat
disp('adjusted observations, d-m');
[lhat_deg lhat_min]
disp('check cond. eqn. (should be zero)');
chk
```

Listing of Matlab output

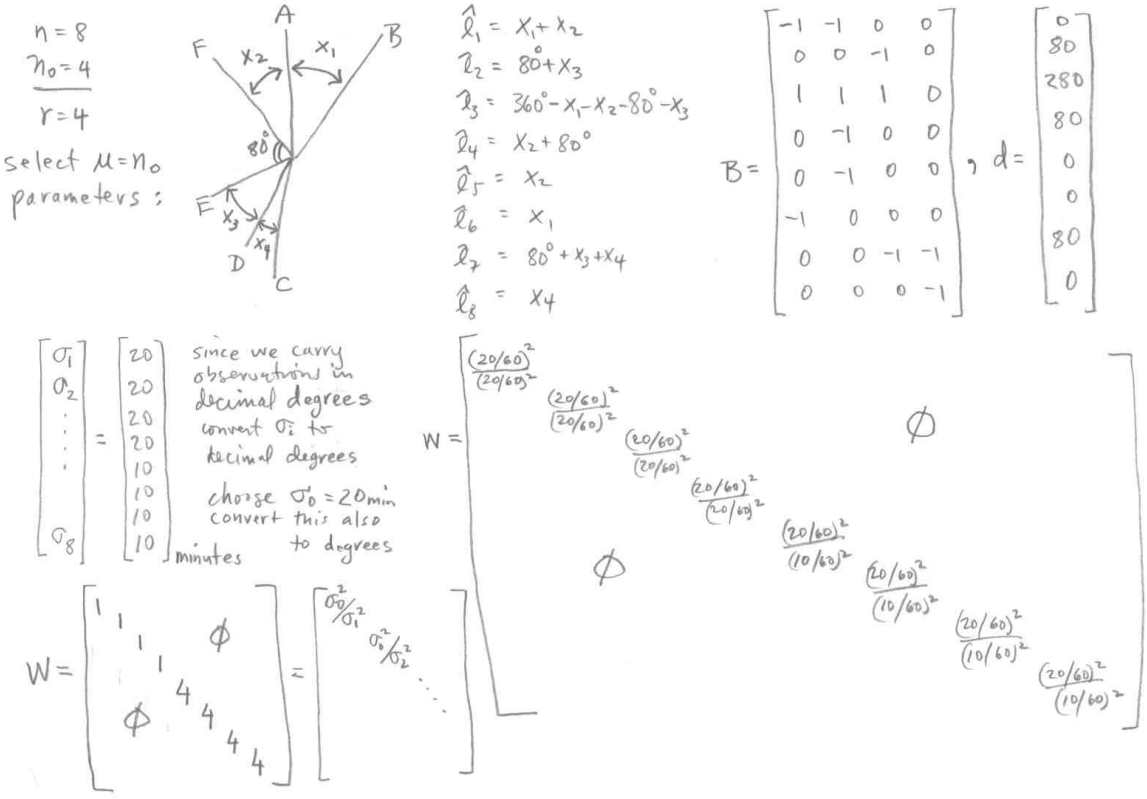
```
hw41a
original observations, decimal degrees
l =
   73.5833
  130.9167
  155.6667
  102.9000
   23.2500
   50.3333
  146.4167
   15.6667
original observations, d-m
ans =
   73.0000   35.0000
  130.0000   55.0000
  155.0000   40.0000
  102.0000   54.0000
   23.0000   15.0000
   50.0000   20.0000
  146.0000   25.0000
   15.0000   40.0000
weight matrix
W =
     1     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0
     0     0     1     0     0     0     0     0
     0     0     0     1     0     0     0     0
     0     0     0     0     1     0     0     0
     0     0     0     0     0     1     0     0
     0     0     0     0     0     0     1     0
     0     0     0     0     0     0     0     1
```

```
A matrix
A =
     1     1     1     0     0     0     0     0
     0     1     0     0     0     0    -1     1
    -1     0     0     0     1     1     0     0
     0     0     0     1    -1     0     0     0
f vector
f =
   -0.1667
   -0.1667
   -0.0000
    0.3500
residuals, decimal degrees
v =
   -0.0809
   -0.0676
   -0.0181
    0.2064
   -0.1436
    0.0627
    0.0495
   -0.0495
residuals, d=m
v_min =
   -4.8529
   -4.0588
   -1.0882
   12.3824
   -8.6176
    3.7647
    2.9706
   -2.9706
lagrange multipliers
k =
   -0.0181
   -0.0495
    0.0627
    0.2064
adjusted observations, decimal degrees
lhat =
   73.5025
  130.8490
  155.6485
  103.1064
   23.1064
   50.3961
  146.4662
   15.6172
```

```
adjusted observations, d-m
ans =
   73.0000   30.1471
  130.0000   50.9412
  155.0000   38.9118
  103.0000    6.3824
   23.0000    6.3824
   50.0000   23.7647
  146.0000   27.9706
   15.0000   37.0294
check cond. eqn. (should be zero)
chk =
  1.0e-016 *

         0
         0
    0.2776
         0
diary off
```

Problem 1(b) angle figure, indirect observations, use the given sigmas



Matlab code

```
% hw41b.m  ce506 - '03
% 27-sep-03

% set up the problem
% solve by indirect observations

n=8;
n0=4;
r=4;
c=n;

sig_min=[20;20;20;20;10;10;10;10];
sig=sig_min/60;
sig0=max(sig);

l=[73+35/60; 130+55/60; 155+40/60; 102+54/60;
   23+15/60; 50+20/60; 146+25/60; 15+40/60];
W=diag(sig0^2./sig.^2);
B=[-1 -1 0 0;
   0 0 -1 0;
   1 1 1 0;
   0 -1 0 0;
   0 -1 0 0;
   -1 0 0 0;
   0 0 -1 -1;
   0 0 0 -1];
d=[0;80;280;80;0;0;80;0];
f=d-l;

% solution

del=inv(B'*W*B)*B'*W*f;
v=f-B*del;
lhat=l+v;

l_deg=fix(l);
l_min=(l-l_deg)*60;
lhat_deg=fix(lhat);
lhat_min=(lhat-lhat_deg)*60;
v_min=v*60;

disp('original observations, decimal degrees');
l
disp('original observations, d-m');
[l_deg l_min]
disp('weight matrix');
W
disp('B matrix');
B
disp('f vector');
f
disp('residuals, decimal degrees');
v
disp('residuals, d=m');
```

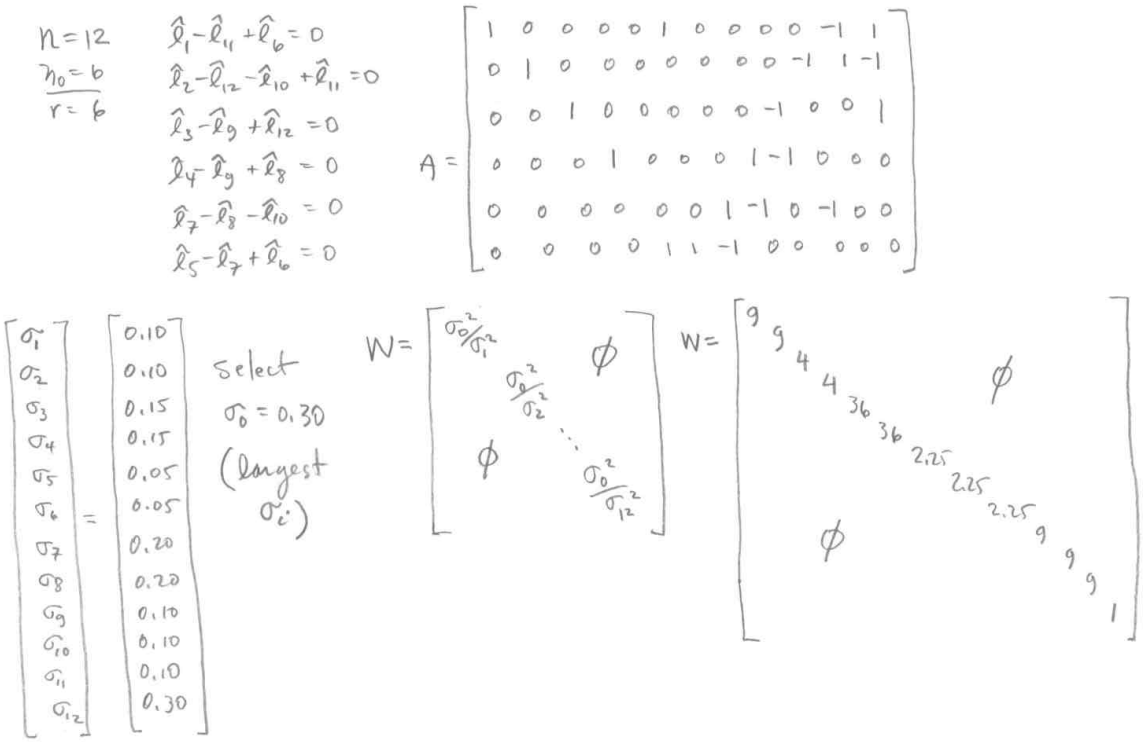
```
v_min
disp('parameters');
del
disp('adjusted observations, decimal degrees');
lhat
disp('adjusted observations, d-m');
[lhat_deg lhat_min]
```

Matlab output listing

```
hw41b
original observations, decimal degrees
l =
   73.5833
  130.9167
  155.6667
  102.9000
   23.2500
   50.3333
  146.4167
   15.6667
original observations, d-m
ans =
   73.0000   35.0000
  130.0000   55.0000
  155.0000   40.0000
  102.0000   54.0000
   23.0000   15.0000
   50.0000   20.0000
  146.0000   25.0000
   15.0000   40.0000
weight matrix
W =
     1     0     0     0     0     0     0     0
     0     1     0     0     0     0     0     0
     0     0     1     0     0     0     0     0
     0     0     0     1     0     0     0     0
     0     0     0     0     4     0     0     0
     0     0     0     0     0     4     0     0
     0     0     0     0     0     0     4     0
     0     0     0     0     0     0     0     4
B matrix
B =
    -1    -1     0     0
     0     0    -1     0
     1     1     1     0
     0    -1     0     0
     0    -1     0     0
    -1     0     0     0
     0     0    -1    -1
     0     0     0    -1
```

```
f vector
f =
  -73.5833
  -50.9167
  124.3333
  -22.9000
  -23.2500
  -50.3333
  -66.4167
  -15.6667
residuals, decimal degrees
v =
   -0.0497
   -0.1126
   -0.0044
    0.2890
   -0.0610
    0.0113
    0.0270
   -0.0270
residuals, d=m
v_min =
   -2.9790
   -6.7552
   -0.2657
   17.3427
   -3.6573
    0.6783
    1.6224
   -1.6224
parameters
del =
   50.3446
   23.1890
   50.8041
   15.6396
adjusted observations, decimal degrees
lhat =
   73.5337
  130.8041
  155.6622
  103.1890
   23.1890
   50.3446
  146.4437
   15.6396
adjusted observations, d-m
ans =
   73.0000   32.0210
  130.0000   48.2448
  155.0000   39.7343
  103.0000   11.3427
   23.0000   11.3427
   50.0000   20.6783
  146.0000   26.6224
   15.0000   38.3776
diary off
```

Problem 2(a) level network, use observations only, use given sigmas



Matlab code

```
% hw42a.m  27-sep-03
% ce506 homework 4, #2(a)

% solve by observations only, use the sigmas

n=12;
n0=6;
r=n-n0;
c=r;

l=[10.04;1.90;3.12;7.85;7.19;4.92;12.25;1.88;10.18;9.83;15.31;6.92];
sig=[0.1;0.1;0.15;0.15;0.05;0.05;0.20;0.20;0.10;0.10;0.10;0.30];
sig0=max(sig);
W=diag(sig0^2./sig.^2);
A=[1 0 0 0 0 1 0 0 0 0 -1 0;
   0 1 0 0 0 0 0 0 0 -1 1 -1;
   0 0 1 0 0 0 0 0 -1 0 0 1;
   0 0 0 1 0 0 0 1 -1 0 0 0;
   0 0 0 0 0 0 1 -1 0 -1 0 0;
   0 0 0 0 1 1 -1 0 0 0 0 0];
f=-A*l;

% build the full normal equations
% and solve
```

```
Mtx=[W -A'; -A zeros(c,c)];
Vec=[zeros(n,1); -f];
Sol=inv(Mtx)*Vec;
v=Sol(1:n);
k=Sol(n+1:n+c);
lhat=l+v;
chk=A*v - f;

disp('original observations');
l
disp('weight matrix');
W
disp('A matrix');
A
disp('f vector');
f
disp('residuals');
v
disp('lagrange multipliers');
k
disp('adjusted observations');
lhat
disp('check cond. eqn. (should be zero)');
chk
```

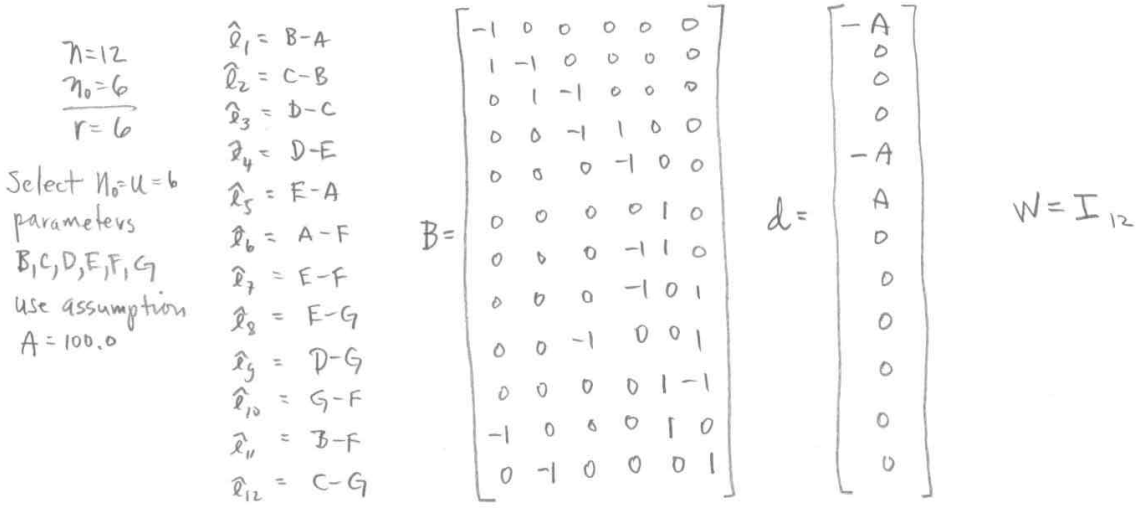
Matlab output listing

```
hw42a
original observations
l =
   10.0400
    1.9000
    3.1200
    7.8500
    7.1900
    4.9200
   12.2500
    1.8800
   10.1800
    9.8300
   15.3100
    6.9200
```

```
weight matrix
W =
  Columns 1 through 7
    9.0000         0         0         0         0         0         0
         0    9.0000         0         0         0         0         0
         0         0    4.0000         0         0         0         0
         0         0         0    4.0000         0         0         0
         0         0         0         0   36.0000         0         0
         0         0         0         0         0   36.0000         0
         0         0         0         0         0         0    2.2500
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
  Columns 8 through 12
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
         0         0         0         0         0
    2.2500         0         0         0         0
         0    9.0000         0         0         0
         0         0    9.0000         0         0
         0         0         0    9.0000         0
         0         0         0         0    1.0000
A matrix
A =
   1     0     0     0     0     1     0     0     0     0    -1     0
   0     1     0     0     0     0     0     0     0    -1     1    -1
   0     0     1     0     0     0     0     0    -1     0     0     1
   0     0     0     1     0     0     0     1    -1     0     0     0
   0     0     0     0     0     0     1    -1     0    -1     0     0
   0     0     0     0     1     1    -1     0     0     0     0     0
f vector
f =
    0.3500
   -0.4600
    0.1400
    0.4500
   -0.5400
    0.1400
```

```
residuals
v =
    0.1413
   -0.0366
   -0.0420
    0.0873
   -0.0046
    0.0307
   -0.1139
    0.3426
   -0.0201
    0.0835
   -0.1780
    0.1619
lagrange multipliers
k =
    1.2718
   -0.3298
   -0.1680
    0.3491
   -0.4218
   -0.1656
adjusted observations
lhat =
   10.1813
    1.8634
    3.0780
    7.9373
    7.1854
    4.9507
   12.1361
    2.2226
   10.1599
    9.9135
   15.1320
    7.0819
check cond. eqn. (should be zero)
chk =
  1.0e-016 *
         0
    0.5551
         0
         0
         0
         0
diary off
```

Problem 2(b) level network, indirect observations, equal precision



Matlab code

```
% hw42b.m  27-sep-03
% ce506 homework 4, #2(b)

% solve by indirect observations, use equal precision assumption

n=12;
n0=6;
r=n-n0;
c=n;

l=[10.04;1.90;3.12;7.85;7.19;4.92;12.25;1.88;10.18;9.83;15.31;6.92];
W=eye(n);
A=100.0;

B=[-1 0 0 0 0 0;
   1 -1 0 0 0 0;
   0 1 -1 0 0 0;
   0 0 -1 1 0 0;
   0 0 0 -1 0 0;
   0 0 0 0 1 0;
   0 0 0 -1 1 0;
   0 0 0 -1 0 1;
   0 0 -1 0 0 1;
   0 0 0 0 1 -1;
   -1 0 0 0 1 0;
   0 -1 0 0 0 1];
d=[-A;0;0;0;-A;A;0;0;0;0;0;0];
f=d - l;

% solve the normal equations

del=inv(B'*W*B)*B'*W*f;
v=f - B*del;
```

```
lhat=l+v;

disp('original observations');
l
disp('weight matrix');
W
disp('B matrix');
B
disp('f vector');
f
disp('parameter vector');
del
disp('residuals');
v
disp('adjusted observations');
lhat
```

Matlab output listing

```
hw42b
original observations
l =
   10.0400
    1.9000
    3.1200
    7.8500
    7.1900
    4.9200
   12.2500
    1.8800
   10.1800
    9.8300
   15.3100
    6.9200
weight matrix
W =
   1     0     0     0     0     0     0     0     0     0     0     0
   0     1     0     0     0     0     0     0     0     0     0     0
   0     0     1     0     0     0     0     0     0     0     0     0
   0     0     0     1     0     0     0     0     0     0     0     0
   0     0     0     0     1     0     0     0     0     0     0     0
   0     0     0     0     0     1     0     0     0     0     0     0
   0     0     0     0     0     0     1     0     0     0     0     0
   0     0     0     0     0     0     0     1     0     0     0     0
   0     0     0     0     0     0     0     0     1     0     0     0
   0     0     0     0     0     0     0     0     0     1     0     0
   0     0     0     0     0     0     0     0     0     0     1     0
   0     0     0     0     0     0     0     0     0     0     0     1
```

B matrix
B =

-1	0	0	0	0	0
1	-1	0	0	0	0
0	1	-1	0	0	0
0	0	-1	1	0	0
0	0	0	-1	0	0
0	0	0	0	1	0
0	0	0	-1	1	0
0	0	0	-1	0	1
0	0	-1	0	0	1
0	0	0	0	1	-1
-1	0	0	0	1	0
0	-1	0	0	0	1

f vector
f =

-110.0400
-1.9000
-3.1200
-7.8500
-107.1900
95.0800
-12.2500
-1.8800
-10.1800
-9.8300
-15.3100
-6.9200

parameter vector
del =

110.1507
111.9964
115.1096
107.1535
95.0058
105.0288

residuals
v =

0.1107
-0.0543
-0.0068
0.1060
-0.0365
0.0742
-0.1022
0.2447
-0.0992
0.1930
-0.1651
0.0476

```
adjusted observations
lhat =
   10.1507
    1.8457
    3.1132
    7.9560
    7.1535
    4.9942
   12.1478
    2.1247
   10.0808
   10.0230
   15.1449
    6.9676
diary off
```