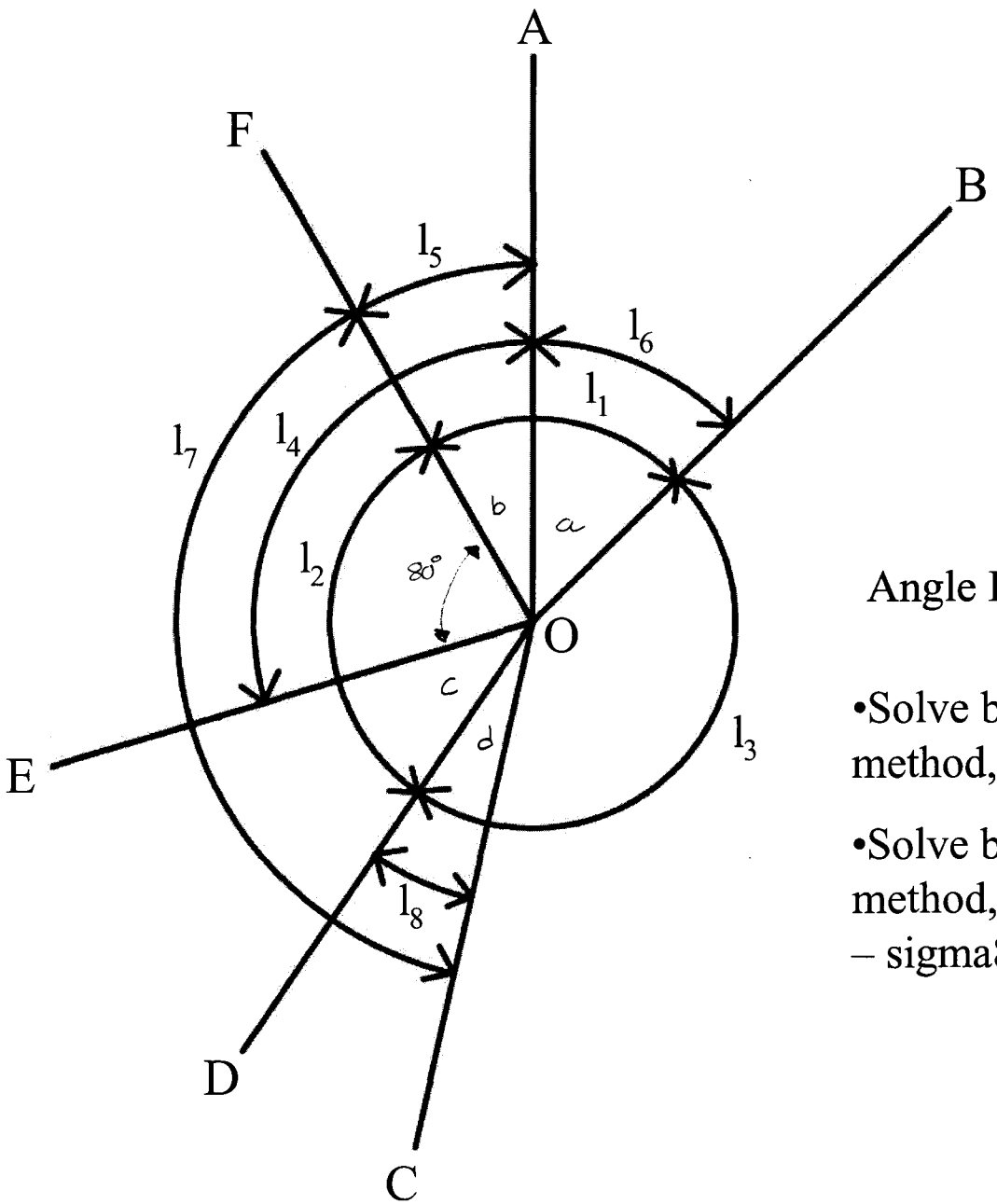
Homework 4, problem 1, Angle figure.



Observation	Value (D-M)
1	73-35
2	130-55
3	155-40
4	102-54
5	23-15
6	50-20
7	146-25
8	15-40

Angle EOF is fixed at 80 degrees.

- Solve by observations only – matrix method, all observations equal precision
- Solve by indirect observations – matrix method, sigma1 – sigma4 = 20 min, sigma5 – sigma8 = 10 min.

1) Indirect observations Method

$n = 8 \rightarrow$ need 8 condition eqns
$n_o = 4 \rightarrow$ need 4 parameters (a, b, c, d)
$r = 4$

Condition Equations

① $\hat{\ell}_1 = a + b$
② $\hat{\ell}_2 = 80 + c$
③ $\hat{\ell}_3 = 360 - 80 - a - b - c$
④ $\hat{\ell}_4 = 80 + b$
⑤ $\hat{\ell}_5 = b$
⑥ $\hat{\ell}_6 = a$
⑦ $\hat{\ell}_7 = 80^\circ + c + d$
⑧ $\hat{\ell}_8 = d$

Add residuals

① $v_1 + \ell_1 = a + b$
② $v_2 + \ell_2 = 80 + c$
③ $v_3 + \ell_3 = 360 - 80 - a - b - c$
④ $v_4 + \ell_4 = 80 + b$
⑤ $v_5 + \ell_5 = b$
⑥ $v_6 + \ell_6 = a$
⑦ $v_7 + \ell_7 = 80^\circ + c + d$
⑧ $v_8 + \ell_8 = d$

← v_s, parameters obs →

① $v_1 - a - b = -\ell_1$
② $v_2 - c = 80 - \ell_2$
③ $v_3 + a + b + c = 360 - 80 - \ell_3$
④ $v_4 - b = 80 - \ell_4$
⑤ $v_5 - b = -\ell_5$
⑥ $v_6 - a = -\ell_6$
⑦ $v_7 - c - d = 80 - \ell_7$
⑧ $v_8 - d = -\ell_8$

Matrix form $V + B\Delta = f$ $\quad f = d - \ell$

$$\begin{bmatrix} v_1 \\ \vdots \\ v_8 \end{bmatrix} + \begin{bmatrix} a & b & c & d \\ -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 280 \\ 80 \\ 0 \\ 0 \\ 80 \\ 0 \end{bmatrix} - \begin{bmatrix} \ell_1 \\ \\ \\ \\ \\ \\ \\ \ell_8 \end{bmatrix}$$

obs in decimal form

$V + B\Delta = f$

Weight Matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & & \phi \\ & \sigma_2^2 & \\ \phi & & \ddots \; \sigma_n^2 \end{bmatrix} = \begin{bmatrix} \sigma_1^2 & & \phi \\ & \ddots & \\ \phi & & \sigma_8^2 \end{bmatrix}$$

Weight Matrix

$$W = \sigma_0^2 \Sigma^{-1} \qquad \sigma_0^2 = 1$$

Delta

$$\Delta = (B^T W B)^{-1}(B^T W f) = N^{-1} t$$

$$\Delta = \begin{bmatrix} 50.3446^\circ \\ 23.1890^\circ \\ 50.8041^\circ \\ 15.6396^\circ \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \quad \text{parameters}$$

Compute Residuals

$$v = f - Ba = \begin{bmatrix} v_1 \\ \vdots \\ v_8 \end{bmatrix}$$

Compute adjusted obs

$$\hat{l}_1 = l_1 + v_1$$

$$\vdots$$

$$\hat{l}_8 = l_1 + v_8$$

Plug parameters back into the condition eqns for a check

(1) $\hat{l}_1 - a - b = -3.5527 e^{-15}$

(2) $\hat{l}_2 - 80 - c = -1.4211 e^{-14}$

(3) $\hat{l}_3 = 360 - 80 - a - b - c$

(4) $\hat{l}_4 - 80 - b = -3.55 e^{-15}$

(5)

(6)

(7)

(8)

1) Observations only Method, Matrix Method

$n = 8$

$n_0 = 4$

$r = 4$ need 4 condition eqns

Condition Eqns

(1) $\hat{\ell}_1 + \hat{\ell}_2 + \hat{\ell}_3 = 360^\circ$

(2) $\hat{\ell}_3 + \hat{\ell}_6 + \hat{\ell}_5 + \hat{\ell}_7 - \hat{\ell}_8 = 360^\circ \Rightarrow$

(3) $\hat{\ell}_4 - \hat{\ell}_5 = 80^\circ$

(4) $\hat{\ell}_1 - \hat{\ell}_5 - \hat{\ell}_6 = 0$

$A\hat{\ell} = d$

$$\underset{4\times8}{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \end{bmatrix}} \underset{8\times1}{\begin{bmatrix} \hat{\ell}_1 \\ \vdots \\ \hat{\ell}_8 \end{bmatrix}} = \begin{bmatrix} 360 \\ 360 \\ 80 \\ 0 \end{bmatrix}$$

Separate $\hat{\ell}$ into $\ell + v$

$$\underset{4\times8}{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \end{bmatrix}} \underset{8\times1}{\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix}} = \underset{4\times1}{\begin{bmatrix} 360 \\ 360 \\ 80 \\ 0 \end{bmatrix}} - \underset{4\times8}{\begin{bmatrix} A \end{bmatrix}} \underset{8\times1}{\begin{bmatrix} 73.58 \\ 130.92 \\ 155.67 \\ 102.90 \\ 23.25 \\ 50.33 \\ 146.42 \\ 15.67 \end{bmatrix}} = \underset{4\times1}{\begin{bmatrix} -0.1667 \\ 0.0000 \\ 0.3500 \\ 0.0000 \end{bmatrix}}$$

$$A \qquad v = d - A\ell = f$$

Matrix Solution

$$\begin{bmatrix} -W & A^T \\ A & \phi \end{bmatrix} \begin{bmatrix} V \\ K \end{bmatrix} = \begin{bmatrix} \phi \\ f \end{bmatrix}$$

$W = \underline{T} = 8\times8$

$A = 4\times8 \quad A^T = 8\times4$

$8\times8 \quad 8\times4$

1)

$$\begin{bmatrix} -1 & & & & & & & & \phi & 1 & 0 & 0 & 1 \\ & -1 & & & & & & & & 1 & 0 & 0 & 0 \\ & & -1 & & & & & & & 1 & 1 & 0 & 0 \\ & & & -1 & & & & & & 0 & 0 & 1 & 0 \\ & & & & -1 & & & & & 0 & 1 & -1 & -1 \\ & & & & & -1 & & & & 0 & 1 & 0 & -1 \\ & & & & & & -1 & & & 0 & 1 & 0 & 0 \\ \phi & & & & & & & -1 & & 0 & -1 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 & -1 & & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -0.1667 \\ 0.0000 \\ 0.3500 \\ 0.0000 \end{bmatrix}$$

12x12 12x1 12x1

$Mx = q$

$x = inv(M) * q$

Solution

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} = \begin{bmatrix} -0.0808 \\ -0.0676 \\ -0.0181 \\ 0.2063 \\ -0.1436 \\ 0.0627 \\ 0.0495 \\ -0.0495 \end{bmatrix}$$

adjusted observations

$\hat{l}_1 = l_1 + v_1$ $\quad \hat{l}_1 = 73.502$

$\hat{l}_8 = l_8 + v_8$ $\quad \hat{l}_8 = 15.617$

$$\begin{bmatrix} 130.850 \\ 155.650 \\ 103.110 \\ 23.106 \\ 50.396 \\ 146.470 \end{bmatrix}$$

Checks

① $360 - \hat{l}_1 - \hat{l}_2 - \hat{l}_3 = \phi$ ✓

② $360 - \hat{l}_3 - \hat{l}_6 - \hat{l}_5 - \hat{l}_7 + \hat{l}_8 = -5e{-}14$ ✓

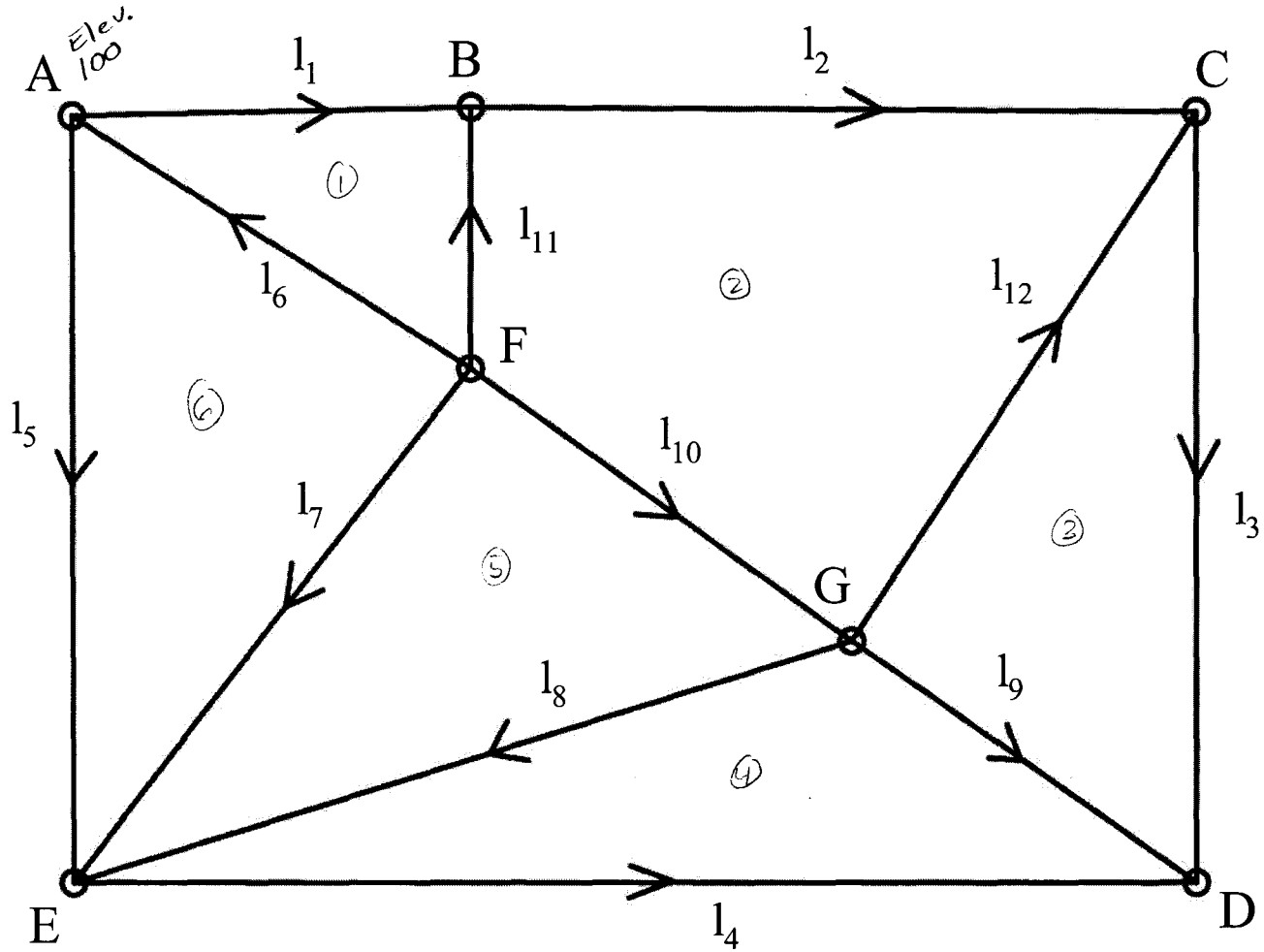
③ $80 - \hat{l}_4 + \hat{l}_5 = 3.5e^{-15}$ ✓

④ $\hat{l}_1 - \hat{l}_5 - \hat{l}_6 = \phi$ ✓

Problem 2. level network

•Solve by observations only using the given sigmas

•Solve by indirect observations assuming equal precision, for this part you may assume that point A has elevation 100.00



Observ.	Value	Sigma
1	10.04	0.10
2	1.90	0.10
3	3.12	0.15
4	7.85	0.15
5	7.19	0.05
6	4.92	0.05
7	12.25	0.20
8	1.88	0.20
9	10.18	0.10
10	9.83	0.10
11	15.31	0.10
12	6.92	0.30

2) Indirect obs method $V + B\Delta = f$ $f = d - \ell$

$n = 12$ need 12 condition eqns
$n_0 = 6$ need 6 parameters (B, C, D, E, F, G)
$r = 6$

Condition Equations	add residuals	← vs, parameters obs →
(1) $\hat{\ell}_1 = B - 100$	(1) $\ell_1 + v_1 = B - 100$	(1) $v_1 - B = -100 - \ell_1$
(2) $\hat{\ell}_2 = C - B$		(2) $v_2 - C + B = -\ell_2$
(3) $\hat{\ell}_3 = D - C$		(3) $v_3 + C - D = -\ell_3$
(4) $\hat{\ell}_4 = D - E$		(4) $v_4 - D + E = -\ell_4$
(5) $\hat{\ell}_5 = E - 100$		(5) $v_5 - E = -100 - \ell_5$
(6) $\hat{\ell}_6 = 100 - F$		(6) $v_6 + F = 100 - \ell_6$
(7) $\hat{\ell}_7 = E - F$		(7) $v_7 - E + F = -\ell_7$
(8) $\hat{\ell}_8 = E - G$		(8) $v_8 - E + G = -\ell_8$
(9) $\hat{\ell}_9 = D - G$		(9) $v_9 - D + G = -\ell_9$
(10) $\hat{\ell}_{10} = G - F$		(10) $v_{10} - G + F = -\ell_{10}$
(11) $\hat{\ell}_{11} = B - F$		(11) $v_{11} - B + F = -\ell_{11}$
(12) $\hat{\ell}_{12} = C - G$	(12) $\ell_{12} + v_{12} = C - G$	(12) $v_{12} - C + G = -\ell_{12}$

Matrix Form $V + B\Delta = f$

$$\begin{bmatrix} v_1 \\ \vdots \\ v_{12} \end{bmatrix}_{12\times1} + \begin{array}{c} \begin{matrix} B & C & D & E & F & G \end{matrix} \\ \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}_{12\times6} \end{array} \begin{bmatrix} B \\ C \\ D \\ E \\ F \\ G \end{bmatrix}_{6\times1} = \begin{bmatrix} -100 \\ 0 \\ 0 \\ 0 \\ -100 \\ 100 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{12\times1} - \begin{bmatrix} 10\times04 \\ 1\times90 \\ 3\times12 \\ 7\times85 \\ 7\times19 \\ 4\times92 \\ 12\times25 \\ 1\times88 \\ 10\times18 \\ 9\times83 \\ 15\times31 \\ 6\times92 \end{bmatrix}_{12\times1}$$

Delta

$$\Delta = (B^T W B)^{-1} \cdot B^T W f$$

$$\begin{bmatrix} B \\ C \\ D \\ E \\ F \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$

Compute Residuals

$$v = f - B\Delta$$

$$\begin{bmatrix} v_1 \\ \vdots \\ v_2 \end{bmatrix} =$$

Compute adj. obs

$$\hat{l}_1 = v_1 + l_1$$

$$\hat{l}_{12} = v_{12} + l_{12}$$

checks

① $\hat{l}_1 - B + 100 = \phi$ ✓

ϕ inbetween

⑫ $\hat{l}_{12} - C + G = \phi$ ✓

```
delta =

  110.1507
  111.9964
  115.1096
  107.1535
   95.0058
  105.0288


BB =

  110.1507


C =

  111.9964


D =

  115.1096


E =

  107.1535


F =

   95.0058


G =

  105.0288


v =

    0.1107
   -0.0543
   -0.0068
    0.1060
   -0.0365
    0.0742
   -0.1022
    0.2447
   -0.0992
    0.1930
   -0.1651
    0.0476


l_hat =
```

```
    10.1507
     1.8457
     3.1132
     7.9560
     7.1535
     4.9942
    12.1478
     2.1247
    10.0808
    10.0230
    15.1449
     6.9676

check1 =

     0

check2 =

     0

check3 =

     0

check4 =

     0

check5 =

     0

check6 =

     0

check7 =

     0

check8 =

     0

check9 =

     0
```

```
check10 =

     0


check11 =

     0


check12 =

     0

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```

2) observations only with sigmas

$n = 12$
$n_0 = 6$
$r = 6 \Rightarrow$ need 6 condition eqns

Condition Eqns

(1) $\hat{l}_1 - \hat{l}_{11} + \hat{l}_6 = \phi$
(2) $\hat{l}_2 - \hat{l}_{12} - \hat{l}_{10} + \hat{l}_{11} = \phi$
(3) $\hat{l}_3 - \hat{l}_9 + \hat{l}_{12} = \phi$
(4) $\hat{l}_9 - \hat{l}_4 - \hat{l}_8 = \phi$
(5) $\hat{l}_{10} + \hat{l}_8 - \hat{l}_7 = \phi$
(6) $\hat{l}_5 - \hat{l}_7 + \hat{l}_6 = \phi$

$A\hat{l} = d$

$$\begin{bmatrix} 1 & & & & & 1 & & & & & -1 & \\ & 1 & & & & \phi & & & -1 & 1 & -1 & \\ & & 1 & & & & & -1 & & & 1 & \\ & & & -1 & & & -1 & 1 & & & & \\ & & \phi & & & -1 & 1 & & 1 & & & \phi \\ & & & & 1 & 1 & -1 & & & & & \end{bmatrix} \begin{bmatrix} \hat{l}_1 \\ \\ \\ \\ \\ \hat{l}_{12} \end{bmatrix} = \begin{bmatrix} \phi \\ \phi \\ \phi \\ \phi \\ \phi \\ \phi \end{bmatrix}$$

6×12 A $\quad 12 \times 1$ $\hat{l}$ $\quad 6 \times 1$ d

Separate $\hat{l}$ into $l + v$

$$\underset{6\times12}{\begin{bmatrix} A \end{bmatrix}} \underset{12\times1}{\begin{bmatrix} v_1 \\ \\ \\ v_{12} \end{bmatrix}} = \begin{bmatrix} \phi \\ \phi \\ \phi \\ \phi \\ \phi \\ \phi \end{bmatrix} - \underset{6\times12}{\begin{bmatrix} A \end{bmatrix}} \begin{bmatrix} 10\times04 \\ 1\times90 \\ 3\times12 \\ 7\times85 \\ 7\times19 \\ 4\times92 \\ 12\times25 \\ 1\times88 \\ 10\times18 \\ 9\times83 \\ 15\times31 \\ 6\times92 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \\ \\ \end{bmatrix}$$

$A \cdot v = d - A\,l = f$ (d: 6×1; l: 12×1; f: 6×1)

Weight Matrix

$$\Sigma = \begin{bmatrix} \sigma_1^2 & & \\ & \ddots & \\ & & \sigma_n^2 \end{bmatrix} \qquad W = \sigma_0^2 W^{-1}$$

Matrix form

$$\begin{bmatrix} -w & A^T \\ A & \phi \end{bmatrix} \begin{bmatrix} v \\ k \end{bmatrix} = \begin{bmatrix} \phi \\ f \end{bmatrix}$$

$M \quad x = q$

$Mx = q$
$x = M^{-1}q$

12×12 $\quad 12\times6$
6×12 $\quad 6\times6$

$$X = \begin{bmatrix} v_1 \\ \vdots \\ v_{12} \\ K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \\ K_6 \end{bmatrix}$$

adj. obs

$$\hat{l}_1 = l_1 + v_1$$

$$\vdots$$

$$\hat{l}_{12} = l_{12} + v_{12}$$

checks

① $\hat{l}_1 - \hat{l}_{11} + \hat{l}_6 = -8.8e^{-16}$ ✓

$\vdots$

⑥

```
f =

    0.3500
   -0.4600
    0.1400
   -0.4500
    0.5400
    0.1400


x =

    0.1413
   -0.0366
   -0.0420
    0.0873
   -0.0046
    0.0307
   -0.1139
    0.3426
   -0.0201
    0.0835
   -0.1780
    0.1619
   14.1311
   -3.6650
   -1.8664
   -3.8785
    4.6867
   -1.8399

v =

    0.1413
   -0.0366
   -0.0420
    0.0873
   -0.0046
    0.0307
   -0.1139
    0.3426
   -0.0201
    0.0835
   -0.1780
    0.1619


l_hat =

   10.1813
    1.8634
    3.0780
    7.9373
    7.1854
    4.9507
   12.1361
```

```
    2.2226
   10.1599
    9.9135
   15.1320
    7.0819


check1 =

 -8.8818e-016


check2 =

     0


check3 =

     0


check4 =

  4.4409e-016


check5 =

  1.7764e-015


check6 =

  8.8818e-016

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```