Derive Least Squares Estimation Formula

$$V + B\Delta = f \qquad \text{linearized condition equations}$$

$$V = f - B\Delta$$

now we want to minimize $V^T W V$ (that is the least square objective function — minimize weighted sum of squares of residuals)

$$V^T W V = (f - B\Delta)^T W (f - B\Delta)$$

$$= f^T W f + \Delta^T B^T W B \Delta - \Delta^T B^T W f - f^T W B \Delta$$

These expressions are all <u>scalars</u> so we can transpose & it is same

$$= f^T W f + \Delta^T B^T W B \Delta - f^T W B \Delta - f^T W B \Delta$$

$$W \text{ is symmetric}$$

$$= f^T W f + \Delta^T B^T W B \Delta - 2 f^T W B \Delta$$

to minimize we take <u>derivative</u> with respect to $\underline{\Delta}$ and set = zero (see reference for derivative of scalar with respect to vector !)

$$\frac{d}{d\Delta}(expr) = 2\Delta^T B^T W B - 2 f^T W B = 0 \quad (\text{row vector})$$

transpose result and divide by 2

$$B^T W B \Delta - B^T W f = 0 \quad (\text{column vector})$$

$$B^T W B \Delta = B^T W f$$

$$\boxed{\Delta = (B^T W B)^{-1} B^T W f}$$

that is conventional Least Squares estimation formula.