CE 597Z Adj. of Geospatial Observations **Exam 1**

11 Oct 2007, 1 page notes ok

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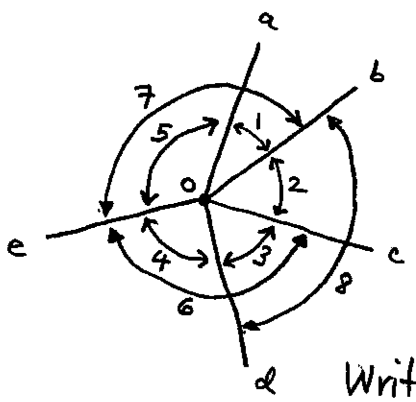
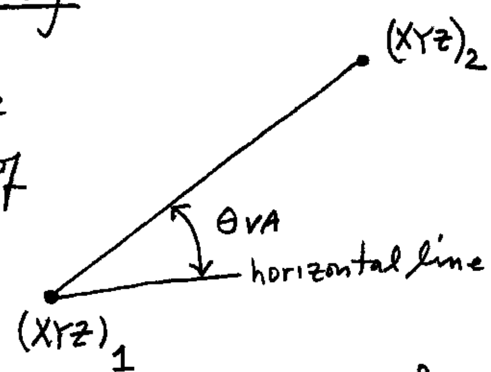
name

1. A weight matrix is given as: $W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$
$\sigma_2 = 0.566$, what is σ_4 ?

2. 

For the angle figure in the sketch, we know that angle $\angle aoc$ is fixed at exactly $90°$. With the given angle observations we wish to adjust the figure. What are n, n_0, and r ? Write the required number of condition equations for the observation only method.

3. 

Write a condition equation for the observed vertical angle, θ_{VA}, in terms of the point coordinates $X_1, Y_1, Z_1, X_2, Y_2, Z_2$.
Assume that we adjust a 3D network by indirect observations, with the point coordinates as unknowns. With $\theta_{VA} = 19.47°$, and with approximations $(X,Y,Z)_1 = (10,10,10)$ and $(XYZ)_2 = (20,20,15)$, evaluate $\frac{\partial F}{\partial Z_1}$ and $f = -F(\ell, x^\circ)$. Give numerical values.

4. A surface is defined by the model $Z = a_0 + a_1 x + a_2 y + a_3 xy$. If 10 points are observed on the surface in all 3 coordinates, and you wish to fit the surface by general least squares, what are n, n_0, and r ?

Useful facts: $\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$, $\frac{d}{dx}\cos^{-1}u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$,

$\frac{d}{dx}\tan^{-1}u = \frac{1}{1+u^2}\frac{du}{dx}$

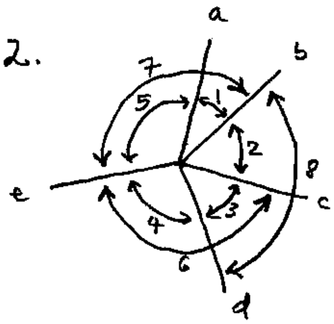
CE 597z Adj. of Geospa. Obs.
Exam 1 Solution 15 Oct 2007

1. $W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ $\quad \sigma_2 = 0.566, \quad \sigma_2^2 = 0.32, \quad w_2 = \sigma_0^2/\sigma_2^2, \quad \sigma_0^2 = \sigma_2^2 w_2$

$\sigma_0^2 = 0.32 \times 2 = 0.64, \quad w_4 = 4 = \sigma_0^2/\sigma_4^2$

$$\sigma_4^2 = \sigma_0^2/4 = \frac{0.64}{4} = 0.16, \quad \underline{\underline{\sigma_4 = 0.4}}$$

2.



$n = 8$
$n_0 = 3$
$r = 5$

for observations only need $c = r = 5$ condition equations

$\hat{l}_1 + \hat{l}_2 + \hat{l}_3 + \hat{l}_4 + \hat{l}_5 = 360°$

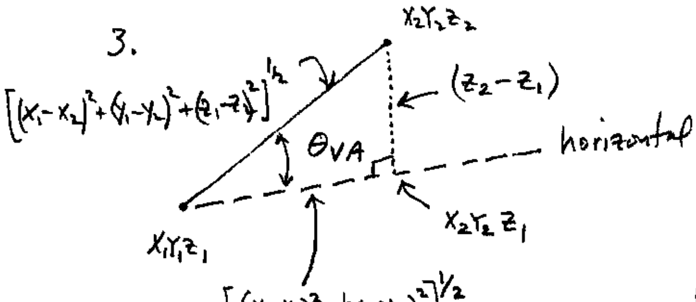
$\hat{l}_1 + \hat{l}_2 = 90°$

$\hat{l}_1 + \hat{l}_5 = \hat{l}_7$

$\hat{l}_3 + \hat{l}_4 = \hat{l}_6$

$\hat{l}_2 + \hat{l}_3 = \hat{l}_8$

3.



$dx = X_2 - X_1 \quad dxy = [dx^2 + dy^2]^{1/2}$
$dy = Y_2 - Y_1 \quad dxyz = [dx^2 + dy^2 + dz^2]^{1/2}$
$dz = Z_2 - Z_1$

$$\theta_{VA} = \tan^{-1} \frac{(Z_2 - Z_1)}{[(X_2-X_1)^2 + (Y_2-Y_1)^2]^{1/2}}$$

(a) $$F_{\theta VA} = \theta_{VA} - \tan^{-1} \frac{(Z_2 - Z_1)}{[(X_2-X_1)^2 + (Y_2-Y_1)^2]^{1/2}} = 0$$

OR

(b) $$F_{\theta VA} = \theta_{VA} - \sin^{-1} \frac{(Z_2 - Z_1)}{[(X_2-X_1)^2 + (Y_2-Y_1)^2 + (Z_2-Z_1)^2]^{1/2}} = 0$$

OR

(c) $$F_{\theta VA} = \theta_{VA} - \cos^{-1} \frac{[(X_2-X_1)^2 + (Y_2-Y_1)^2]^{1/2}}{[(X_2-X_1)^2 + (Y_2-Y_1)^2 + (Z_2-Z_1)^2]^{1/2}} = 0$$

(a) $$\frac{\partial F}{\partial z_1} = \frac{-1}{1+u^2} \cdot \frac{du}{dz_1} = -\frac{1}{1 + \frac{d_z^2}{d_{xy}^2}} \cdot \frac{-1}{d_{xy}} = \frac{1}{d_{xy} + \frac{d_z^2}{d_{xy}}} = \frac{1}{14.142 + \frac{25}{14.142}} = \underline{.0628}$$

$$f = -F(l, x^0) = -(19.47° - \tan^{-1}(\tfrac{5}{14.142})) = \underline{.0014°} = \underline{2.4 \times 10^{-5} \text{ Rad}}$$

(b) $$\frac{\partial F}{\partial z_1} = -\frac{1}{\sqrt{1-u^2}} \cdot \frac{du}{dz_1} = \cancel{-} \frac{1}{\sqrt{1 - \frac{d_z^2}{d_{xyz}^2}}} \cdot \frac{d_{xyz}(\cancel{-}1) - d_z \cdot \cancel{\tfrac{1}{2}}(d_{xyz}^2)^{-1/2} \cdot \cancel{2} d_z(\cancel{-}1)}{d_{xyz}^2}$$

$$= \frac{1}{\sqrt{1 - \frac{d_z^2}{d_{xyz}^2}}} \cdot \frac{d_{xyz}^2 - d_z^2}{d_{xyz}^3} = \frac{1}{\sqrt{1 - \frac{25}{225}}} \cdot \frac{225 - 25}{(15)^3} = \underline{.0628}$$

$$f = -F(l, x^0) = -(19.47 - \sin^{-1}(\tfrac{5}{15})) = \underline{.0012°} = \underline{2.1 \times 10^{-5} \text{ Rad}}$$

$dx = 10 \quad dxy = 14.142$
$dy = 10 \quad dxyz = 15$
$dz = 5$

(c) $$\frac{\partial F}{\partial z_1} = -\frac{-1}{\sqrt{1-u^2}}\,\frac{du}{dz_1} = + \frac{+1}{\sqrt{1-\frac{d_{xy}^2}{d_{xyz}^2}}} \cdot \frac{d_{xyz}\cdot 0 + d_{xy}\cdot\cancel{\tfrac{1}{2}}\left(D_{xyz}^2\right)^{-1/2}\cdot \cancel{2}\cdot d_z \cdot (+1)}{d_{xyz}^2}$$

$$\frac{1}{\sqrt{1-\frac{d_{xy}^2}{d_{xyz}^2}}}\;\frac{d_{xy}\cdot d_z}{d_{xyz}^3} = \frac{1}{\sqrt{1-\frac{200}{225}}}\cdot\frac{14.142\cdot 5}{(15)^3} = \underline{0.0628}$$

$$f = -F(\ell, x^0) = -\left(19.47 - \cos^{-1}\left(\frac{14.142}{15}\right)\right) = \underline{.0028^\circ} = \underline{4.8\times 10^{-5}\ \text{Rad}}$$

(note: with full precision f for all 3 expressions is .0012)

4. $z = a_0 + a_1 x + a_2 y + a_3 xy$, 10 points observed in $x, y, + z$

general LS:

$n = 10\times 3 = 30 \leftarrow n$

$\underline{n_0 = 4 + 2\times 10 = 24} \leftarrow n_0$

$r = 6 \leftarrow r$

$u = 4$ $\quad a_0, a_1, a_2, a_3$

$c = r + u = 10$; one equation per point.