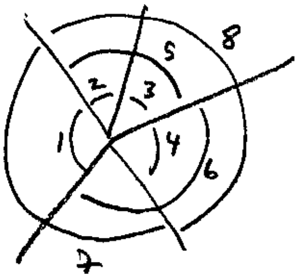
CE597Z Adj. of Geospatial Obs.

HW1 4-Sep → 11-Sep 2007

Solution

1.



$n = 8$ solve by obs. only

$n_0 = 4$ $\Rightarrow c = 4$

$r = 4$

$\hat{\ell}_2 + \hat{\ell}_3 = \hat{\ell}_5$ $\qquad v_2 + v_3 - v_5 = -(\ell_2 + \ell_3 - \ell_5) = 2$

$\hat{\ell}_1 + \hat{\ell}_2 + \hat{\ell}_3 + \hat{\ell}_6 = 360$ $\qquad v_1 + v_2 + v_3 + v_6 = 360 - (\ell_1 + \ell_2 + \ell_3 + \ell_6) = 1$

$\ell_7 + \hat{\ell}_8 = 360$ $\qquad v_7 + v_8 = 360 - (\ell_7 + \ell_8) = -1$

$\hat{\ell}_2 + \hat{\ell}_3 + \hat{\ell}_4 = \hat{\ell}_8$ $\qquad v_2 + v_3 + v_4 - v_8 = -(\ell_2 + \ell_3 + \ell_4 - \ell_8) = 2$

check independence by matrix rank :

$$\begin{bmatrix} 0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

rank = 4 by matlab, so they are independent

$$\Phi' = V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2 + V_6^2 + V_7^2 + V_8^2 + 2K_1(V_2 + V_3 - V_5 - 2) + 2K_2(V_1 + V_2 + V_3 + V_6 - 1) + 2K_3(V_7 + V_8 + 1) + 2K_4(V_2 + V_3 + V_4 - V_8 - 2)$$

$$\frac{\partial \phi'}{\partial V_1} = \cancel{2}V_1 + \cancel{2}K_2 = 0$$

$$\frac{\partial \phi'}{\partial V_2} = \cancel{2}V_2 + \cancel{2}K_1 + \cancel{2}K_2 + \cancel{2}K_4 = 0$$

$$\frac{\partial \phi'}{\partial V_3} = \cancel{2}V_3 + \cancel{2}K_1 + \cancel{2}K_2 + \cancel{2}K_4 = 0$$

$$\frac{\partial \phi'}{\partial V_4} = \cancel{2}V_4 + \cancel{2}K_4 = 0$$

$$\frac{\partial \phi'}{\partial V_5} = \cancel{2}V_5 - \cancel{2}K_1 = 0$$

$$\frac{\partial \phi'}{\partial v_6} = \cancel{2}v_6 + \cancel{2}k_2 = 0$$

$$\frac{\partial \phi'}{\partial v_7} = \cancel{2}v_7 + \cancel{2}k_3 = 0$$

$$\frac{\partial \phi'}{\partial v_8} = \cancel{2}v_8 + \cancel{2}k_3 - \cancel{2}k_4 = 0$$

$$\frac{\partial \phi'}{\partial k_1} = \cancel{2}(v_2+v_3-v_5-2) = 0$$

$$\frac{\partial \phi'}{\partial k_2} = \cancel{2}(v_1+v_2+v_3+v_6-1) = 0$$

$$\frac{\partial \phi'}{\partial k_3} = \cancel{2}(v_7+v_8+1) = 0$$

$$\frac{\partial \phi'}{\partial k_4} = \cancel{2}(v_2+v_3+v_4-v_8-2) = 0$$

$$\begin{array}{c}
\begin{array}{cccccccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 & k_1 & k_2 & k_3 & k_4 \end{array} \\
\left[\begin{array}{cccccccc|cccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 \\
\hline
0 & 1 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \\ 1 \\ -1 \\ 2 \end{bmatrix}$$

normal equations, note structure from partition

Solve system:

$$V = \begin{bmatrix} -.156 \\ .656 \\ .656 \\ .125 \\ -.687 \\ -.156 \\ -.437 \\ -.562 \end{bmatrix} \quad K = \begin{bmatrix} -.687 \\ .156 \\ .437 \\ -.125 \end{bmatrix} \quad \hat{l} = l + v = \begin{bmatrix} 95.844 \\ 44.656 \\ 37.656 \\ 98.125 \\ 82.313 \\ 181.84 \\ 179.56 \\ 180.44 \end{bmatrix}$$

check condition equations satisfied ✓✓✓✓

2. $X + V_x = ax + by + c$, $\quad V_x = ax + by + c - X$

$Y + V_y = -bx + ay + d$, $\quad V_y = -bx + ay + d - Y$

$$V_{x_1} = 1.a + 1.b + c - 5.8$$
$$V_{y_1} = -1.b + 1.a + d - 8.4$$
$$V_{x_2} = 2a + 2b + c - 7.3$$
$$V_{y_2} = -2b + 2a + d - 10.6$$
$$V_{x_3} = 1.a + 2.b + c - 5.5$$
$$V_{y_3} = -1b + 2.a + d - 10.2$$

$n = 6$

$n_0 = 4$

$r = 2$

Solve by indirect observations

choose $\mu = n_0 = 4$ parameters:

a, b, c, d

$$\phi = V_{x_1}^2 + V_{y_1}^2 + V_{x_2}^2 + V_{y_2}^2 + V_{x_3}^2 + V_{y_3}^2 =$$

$$(a + b + c - 5.8)^2 + (-b + a + d - 8.4)^2 + (2a + 2b + c - 7.3)^2 +$$

$$(-2b + 2a + d - 10.6)^2 + (a + 2b + c - 5.5)^2 + (-b + 2a + d - 10.2)^2$$

$$\frac{\partial \phi}{\partial a} = \cancel{2}(a+b+c-5.8) + \cancel{2}(-b+a+d-8.4) + \cancel{2}(2a+2b+c-7.3)\cdot 2 + \cancel{2}(-2b+2a+d-10.6)\cdot 2 + \cancel{2}(a+2b+c-5.5) + \cancel{2}(-b+2a+d-10.2)\cdot 2 = 0$$

$$\frac{\partial \phi}{\partial b} = \cancel{2}(a+b+c-5.8) + \cancel{2}(-b+a+d-8.4)(-1) + \cancel{2}(2a+2b+c-7.3)2 + \cancel{2}(-2b+2a+d-10.6)(-2) + \cancel{2}(a+2b+c-5.5)\cdot 2 + \cancel{2}(-b+2a+d-10.2)(-1) = 0$$

$$\frac{\partial \phi}{\partial c} = \cancel{2}(a+b+c-5.8) + \cancel{2}(2a+2b+c-7.3) + \cancel{2}(a+2b+c-5.5) = 0$$

$$\frac{\partial \phi}{\partial d} = \cancel{2}(-b+a+d-8.4) + \cancel{2}(-2b+2a+d-10.6) + \cancel{2}(-b+2a+d-10.2) = 0$$

$$15a + 0b + 4c + 5d = 5.8 + 8.4 + (7.3)2 + (10.6)2 + 5.5 + (10.2)2$$

$$0a + 15b + 5c + -4d = 5.8 - 8.4 + (7.3)2 - (10.6)2 + (5.5)2 - 10.2$$

$$4a + 5b + 3c + 0d = 5.8 + 7.3 + 5.5$$

$$5a - 4b + 0c + 3d = 8.4 + 10.6 + 10.2$$

$$\begin{bmatrix} 15 & 0 & 4 & 5 \\ 0 & 15 & 5 & -4 \\ 4 & 5 & 3 & 0 \\ 5 & -4 & 0 & 3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 75.9 \\ -8.4 \\ 18.6 \\ 29.2 \end{bmatrix}$$

Solve in matlab

$$\begin{bmatrix} a \\ b \\ c \\ a \end{bmatrix} = \begin{bmatrix} 1.825 \\ -.350 \\ 4.350 \\ 6.225 \end{bmatrix}$$

plug into condition equations for V's :

$V_{x_1} = .025$ $\quad \hat{X}_1 = 5.825$

$V_{y_1} = 0$ $\quad \hat{Y}_1 = 8.4$

$V_{x_2} = 0$ $\quad \hat{X}_2 = 7.3$

$V_{y_2} = -.025$ $\quad \hat{Y}_2 = 10.575$

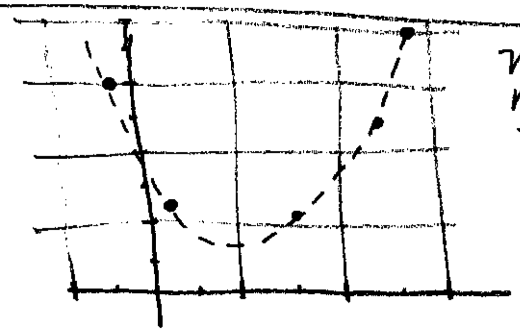
$V_{x_3} = -.025$ $\quad \hat{X}_3 = 5.475$

$V_{y_3} = .025$ $\quad \hat{Y}_3 = 10.225$

check 6 cond. eqn's satisfied :

✓ ✓
✓ ✓
✓ ✓

3.



$n = 5$ use indirect observations

$n_0 = 3$ $\quad u = n_0 = 3$

$r = 2$ choose a_0, a_1, a_2 for the parameters

$c = n$ use condition equations of the form

$$y_i + V_i = a_0 + a_1 x_i + a_2 x_i^2$$

$$V_1 = a_0 + (-0.4)\, a_1 + (.16) a_2 - 3.0$$
$$V_2 = a_0 + (0.2)\, a_1 + (.04) a_2 - 1.2$$
$$V_3 = a_0 + (1.5)\, a_1 + (2.25) a_2 - 1.1$$
$$V_4 = a_0 + (2.4)\, a_1 + (5.76) a_2 - 2.4$$
$$V_5 = a_0 + (2.8)\, a_1 + (7.84) a_2 - 4.0$$

$$\phi = V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2 = (a_0 - .4 a_1 + .16 a_2 - 3.0)^2 + (a_0 + .2 a_1 + .04 a_2 - 1.2)^2 + (a_0 + 1.5 a_1 + 2.25 a_2 - 1.1)^2 + (a_0 + 2.4 a_1 + 5.76 a_2 - 2.4)^2 + (a_0 + 2.8 a_1 + 7.84 a_2 - 4.0)^2$$

$$\frac{\partial \phi}{\partial a_0} = \cancel{2}(a_0 - .4 a_1 + .16 a_2 - 3) + \cancel{2}(a_0 + .2 a_1 + .04 a_2 - 1.2) + \cancel{2}(a_0 + 1.5 a_1 + 2.25 a_2 - 1.1) + \cancel{2}(a_0 + 2.4 a_1 + 5.76 a_2 - 2.4) + \cancel{2}(a_0 + 2.8 a_1 + 7.84 a_2 - 4.0) = 0$$

$$\frac{\partial \phi}{\partial a_1} = \cancel{2}(a_0 - .4 a_1 + .16 a_2 - 3.0)(-.4) + \cancel{2}(a_0 + .2 a_1 + .04 a_2 - 1.2)(0.2) + \cancel{2}(a_0 + 1.5 a_1 + 2.25 a_2 - 1.1)(1.5) + \cancel{2}(a_0 + 2.4 a_1 + 5.76 a_2 - 2.4)(2.4) + \cancel{2}(a_0 + 2.8 a_1 + 7.84 a_2 - 4.0)(2.8) = 0$$

$$\frac{\partial \phi}{\partial a_2} = \cancel{2}(a_0 - .4a_1 + .16a_2 - 3.0)(.16) + \cancel{2}(a_0 + .2a_1 + .04a_2 - 1.2)(.04) +$$

$$\cancel{2}(a_0 + 1.5a_1 + 2.25a_2 - 1.1)(2.25) + \cancel{2}(a_0 + 2.4a_1 + 5.76a_2 - 2.4)(5.76) +$$

$$\cancel{2}(a_0 + 2.8a_1 + 7.84a_2 - 4.0)\ 7.84 = 0$$

$$5a_0 + 6.5a_1 + 16.05a_2 = 11.7$$

$$6.5a_0 + 16.05a_1 + 39.095a_2 = 17.65$$

$$16.05a_0 + 39.095a_1 + 99.733a_2 = 48.187$$

$$\begin{bmatrix} 5 & 6.5 & 16.05 \\ 6.5 & 16.05 & 39.095 \\ 16.05 & 39.095 & 99.733 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 11.7 \\ 17.65 \\ 48.187 \end{bmatrix}, \quad \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1.825 \\ -2.234 \\ 1.065 \end{bmatrix}$$

solve by matlab

$$V = \begin{bmatrix} -0.1109 \\ 0.2207 \\ -0.2295 \\ 0.1988 \\ -0.0791 \end{bmatrix}, \quad \hat{y} = \begin{bmatrix} 2.8891 \\ 1.4207 \\ 0.8705 \\ 2.5988 \\ 3.9209 \end{bmatrix}$$

4. $\overset{l_1}{6.0}, \overset{l_2}{5.8}, \overset{l_3}{6.1}, \overset{l_4}{6.2}$ $n=4, n_0=1, r=3$ $(\bar{x} = 6.025)$

<u>observations only</u> :

$c=3$

$$\hat{l}_1 = \hat{l}_2 , \quad \hat{l}_1 - \hat{l}_2 = 0 , \quad v_1 - v_2 = -(l_1 - l_2) = -(6.0 - 5.8) = -0.2$$

$$\hat{l}_2 = \hat{l}_3 , \quad \hat{l}_2 - \hat{l}_3 = 0 , \quad v_2 - v_3 = -(l_2 - l_3) = -(5.8 - 6.1) = 0.3$$

$$\hat{l}_3 = \hat{l}_4 , \quad \hat{l}_3 - \hat{l}_4 = 0 , \quad v_3 - v_4 = -(l_3 - l_4) = -(6.1 - 6.2) = 0.1$$

$$\Phi = v_1^2 + v_2^2 + v_3^2 + v_4^2 + 2k_1(v_1 - v_2 + 0.2) + 2k_2(v_2 - v_3 - 0.3) + 2k_3(v_3 - v_4 - 0.1)$$

$$\frac{\partial\phi}{\partial v_1} = \cancel{2}v_1 + \cancel{2}k_1 = 0$$

$$\frac{\partial\phi}{\partial v_2} = \cancel{2}v_2 - \cancel{2}k_1 + \cancel{2}k_2 = 0$$

$$\frac{\partial\phi}{\partial v_3} = \cancel{2}v_3 - \cancel{2}k_2 + \cancel{2}k_3 = 0$$

$$\frac{\partial\phi}{\partial v_4} = \cancel{2}v_4 - \cancel{2}k_3 = 0$$

$$\frac{\partial\phi}{\partial k_1} = \cancel{2}(v_1 - v_2 + 0.2) = 0$$

$$\frac{\partial\phi}{\partial k_2} = \cancel{2}(v_2 - v_3 - 0.3) = 0$$

$$\frac{\partial\phi}{\partial k_3} = \cancel{2}(v_3 - v_4 - 0.1) = 0$$

$$\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & k_1 & k_2 & k_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.2 \\ 0.3 \\ 0.1 \end{bmatrix}$$

Solution by MATLAB :

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ \hdashline k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} .025 \\ .225 \\ -.075 \\ -.175 \\ \hdashline -.025 \\ -.125 \\ -.175 \end{bmatrix} , \quad \hat{l} = l + v = \begin{bmatrix} 6 \\ 5.8 \\ 6.1 \\ 6.2 \end{bmatrix} + \begin{bmatrix} .025 \\ .225 \\ -.075 \\ -.175 \end{bmatrix} = \begin{bmatrix} 6.025 \\ 6.025 \\ 6.025 \\ 6.025 \end{bmatrix}$$

✓

$\Rightarrow$ same as sample mean

$$\begin{array}{cccc} \ell_1 & \ell_2 & \ell_3 & \ell_4 \\ 6.0, & 5.8, & 6.1, & 6.2 \end{array}$$

4. 6.0, 5.8, 6.1, 6.2 $n=4,\ n_0=1,\ r=3$ $(\bar{x}=6.025)$

<u>indirect observations</u>

Select $\mu = n_0 = 1$ parameter, call it x, to represent the unknown quantity

$c = n = 4$

$\ell_1+v_1 = x$; $v_1 = x - 6.0$

$\ell_2+v_2 = x$; $v_2 = x - 5.8$

$\ell_3+v_3 = x$; $v_3 = x - 6.1$

$\ell_4+v_4 = x$; $v_4 = x - 6.2$

$$\phi = v_1^2+v_2^2+v_3^2+v_4^2 = (x-6.0)^2+(x-5.8)^2+(x-6.1)^2+(x-6.2)^2$$

$$\frac{\partial\phi}{\partial x} = \cancel{2}(x-6.0) + \cancel{2}(x-5.8) + \cancel{2}(x-6.1) + \cancel{2}(x-6.2) = 0$$

$$4x = 6.0+5.8+6.1+6.2$$

$$x = \frac{6.0+5.8+6.1+6.2}{4} = 6.025$$

So LS estimate is same as sample mean!