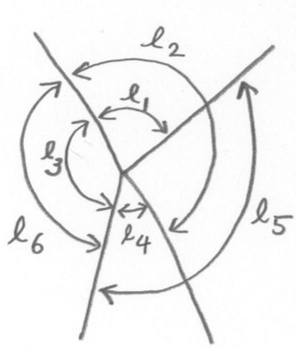
Example: Observations only via matrix methods



#	ℓ
1	100.1
2	215.2
3	114.8
4	29.7
5	145.1
6	115.2

observations are of equal precision and uncorrelated
$\Rightarrow W = I$

$n = 6$
$n_0 = 3$
$r = 3$

we must write $r = 3$ condition equations among the adjusted observations

$$\hat{\ell}_1 + \hat{\ell}_3 + \hat{\ell}_5 = 360^\circ$$
$$\hat{\ell}_2 + \hat{\ell}_3 + \hat{\ell}_4 = 360^\circ \Rightarrow \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \hat{\ell}_1 \\ \hat{\ell}_2 \\ \hat{\ell}_3 \\ \hat{\ell}_4 \\ \hat{\ell}_5 \\ \hat{\ell}_6 \end{bmatrix} = \begin{bmatrix} 360^\circ \\ 360^\circ \\ 0 \end{bmatrix}, \quad \boxed{A\hat{\ell} = d}$$
$$\hat{\ell}_3 - \hat{\ell}_6 = 0$$

Separate $\hat{\ell}$ into $\ell + v$:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = \begin{bmatrix} 360 \\ 360 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 100.1 \\ 215.2 \\ 114.8 \\ 29.7 \\ 145.1 \\ 115.2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.3 \\ 0.4 \end{bmatrix}, \quad \boxed{Av = f}$$

$A \qquad v = d - A \qquad \ell \qquad f$

Now we have all required raw materials for the matrix LS solution

$$\begin{bmatrix} -W & A^T \\ A & 0 \end{bmatrix} \begin{bmatrix} v \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ f \end{bmatrix}$$

$$\left[\begin{array}{cccccc|ccc} -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -1 \\ \hline 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ .3 \\ .4 \end{bmatrix} \qquad \boxed{Mx = q}$$

full solution

in MATLAB, if you define the matrices W, A, f you can create the partitioned full matrices:

M = [-W A' ; A zeros(3,3)] ; q = [zeros(6,1) ; f] ; x = inv(M) * q

or, you can make a solution by elimination, in 2 steps:

$$k = (AQA^T)^{-1} f$$
$$v = QA^T k$$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ \hdashline k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} -.0917 \\ .0583 \\ .1833 \\ .0583 \\ -.0917 \\ -.2167 \\ \hdashline -.0917 \\ .0583 \\ .2167 \end{bmatrix} \qquad \underline{\hat{\ell} = \ell + v}$$

notes: in MATLAB, make a tranpose with apostrophe
$A^T \rightarrow A'$
you can make identity by
W = eye(6)