CE 597 (029) Adj. of Geospatial Obs. HW2 Solution
10 Oct. 2008

1. Observations only ⇒ need $c = r = 5$ condition equations

$n = 10$, $n_0 = 5$, $r = 5$

1. $\hat{l}_1 + \hat{l}_2 + \hat{l}_8 = 180$
2. $\hat{l}_2 + \hat{l}_3 + \hat{l}_4 = 360$
3. $\hat{l}_9 - \hat{l}_8 + \hat{l}_3 + \hat{l}_5 = 180$
4. $\hat{l}_{10} - (\hat{l}_9 - \hat{l}_8) + \hat{l}_6 + \hat{l}_7 = 180$
   $\hat{l}_{10} - \hat{l}_9 + \hat{l}_8 + \hat{l}_6 + \hat{l}_7 = 180$
5. $\hat{l}_8 + \hat{l}_{10} = 180$

$l_1 + v_1 + l_2 + v_2 + l_8 + v_8 = 180$

$l_2 + v_2 + l_3 + v_3 + l_4 + v_4 = 360$

$l_9 + v_9 - l_8 - v_8 + l_3 + v_3 + l_5 + v_5 = 180$

$l_{10} + v_{10} - l_9 - v_9 + l_8 + v_8 + l_6 + v_6 + l_7 + v_7 = 180$

$l_8 + v_8 + l_{10} + v_{10} = 180$

$v_1 + v_2 + v_8 = 180 - (l_1 + l_2 + l_8)$

$v_2 + v_3 + v_4 = 360 - (l_2 + l_3 + l_4)$

$v_9 - v_8 + v_3 + v_5 = 180 - (l_9 - l_8 + l_3 + l_5)$

$v_{10} - v_9 + v_8 + v_6 + v_7 = 180 - (l_{10} - l_9 + l_8 + l_6 + l_7)$

$v_8 + v_{10} = 180 - (l_8 + l_{10})$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \\ v_9 \\ v_{10} \end{bmatrix} = \begin{bmatrix} 180 \\ 360 \\ 180 \\ 180 \\ 180 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \\ l_5 \\ l_6 \\ l_7 \\ l_8 \\ l_9 \\ l_{10} \end{bmatrix}$$

$A \cdot v = d - A \cdot l$, with $f = d - A \cdot l$

$$\boxed{Av = d - Al}$$

$$\boxed{Av = f}$$

choose $\sigma_0 = 2$, $\sigma_0^2 = 4$, $w_i = \sigma_0^2 / \sigma_i^2$

Solution from Matlab:

$k = We\, f$ $\quad We = (AQA^T)^{-1}$, $Q = W^{-1}$, $f = d - Al$

$v = QA^T k$

(from Matlab)

$$k = \begin{bmatrix} 0.475 \\ -1.55 \\ 0.175 \\ 0.725 \\ 0.125 \end{bmatrix}, \quad v = \begin{bmatrix} .119 \\ -.269 \\ -.344 \\ -.388 \\ .044 \end{bmatrix} \rightarrow \begin{bmatrix} .725 \\ .725 \\ 1.15 \\ .55 \\ .85 \end{bmatrix}, \quad \hat{l} = \begin{bmatrix} 61.119 \\ 57.731 \\ 61.656 \\ 240.613 \\ 59.044 \end{bmatrix} \rightarrow \begin{bmatrix} 48.725 \\ 71.725 \\ 61.15 \\ 120.45 \\ 118.85 \end{bmatrix}$$

2. $Z = a_0 + a_1 x + a_2 y + a_3 (x^2 + y^2)$ (paraboloid of revolution)

indirect observations z: obs, x, y: constant

$n = 6$

$n_0 = 4$

$r = 2$

for matrix solution, put in form $V + B\Delta = f$:

$z_i + v_i = a_0 + a_1 x_i + a_2 y_i + a_3 (x_i^2 + y_i^2)$

$v_i - a_0 - a_1 x_i - a_2 y_i - a_3 (x_i^2 + y_i^2) = -z_i$ ←

$$B = \begin{bmatrix} -1 & -5 & -5 & -50 \\ -1 & 5 & -6 & -61 \\ -1 & 4 & 3 & -25 \\ -1 & -4 & 2 & -20 \\ -1 & 2 & -2 & -8 \\ -1 & -1 & 0 & -1 \end{bmatrix}, \quad f = \begin{bmatrix} -4.28 \\ -5.61 \\ -5.47 \\ -4.23 \\ -3.42 \\ -3.23 \end{bmatrix}, \quad c = n = 6, \quad u = n_0 = 4$$

$\sigma_0 = 0.02$, for $\sigma_i = .02$, $w_i = 1$, for $\sigma_i = .01$, $w_i = \frac{(.02)^2}{(.01)^2} = \frac{.0004}{.0001} = 4$

$$W = \begin{bmatrix} 1 & & & 0 \\ & 4 & & \\ & & 1 & \\ 0 & & & 4 \\ & & & & 4 \end{bmatrix}$$

problem is linear so no iterations are required.

$\Delta = (B^T W B)^{-1} B^T W f$, $v = f - B\Delta$

$$\Delta = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 3.2447 \\ -.0970 \\ -.1944 \\ .0498 \end{bmatrix}, \quad V = \begin{bmatrix} -.00008 \\ -.00641 \\ -.00820 \\ .01237 \\ .02859 \\ -.03247 \end{bmatrix}, \quad \hat{l} = \begin{bmatrix} 4.28 \\ 5.60 \\ 5.46 \\ 4.24 \\ 3.45 \\ 3.20 \end{bmatrix}$$

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3. Trilateration, indirect observations — non-linear problem! must iterate solutions

$n = 5$

$n_0 = 2$ x, y

$r = 3$

$c = n = 5$

$u = n_0 = 2$

$$d_i = \left[(x - x_i)^2 + (y - y_i)^2 \right]^{1/2}$$

$$F_i = d_i - \left[(x - x_i)^2 + (y - y_i)^2 \right]^{1/2} = 0$$

$B = \partial F / \partial x$, $f = -F_i$, see graphic for initial Approx $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} 410.5 \\ 462.4 \end{pmatrix}$

$$\frac{\partial F_i}{\partial x} = -\frac{(x - x_i)}{D_i}$$

$$\frac{\partial F_i}{\partial y} = \frac{-(y - y_i)}{D_i}$$

$$B = \begin{bmatrix} .42589 & .90477 \\ .99705 & -.043544 \\ .19971 & -.97986 \\ -.94523 & -.3624 \\ -.77559 & .63124 \end{bmatrix}, \quad f = \begin{bmatrix} -2.4229 \\ -.4799 \\ 2.3474 \\ 1.1803 \\ -1.0864 \end{bmatrix}$$

first iter.

first iter.

$$D_i = \left[(x_0 - x_i)^2 + (y_0 - y_i)^2 \right]^{1/2}$$

$$W = \begin{bmatrix} 7.1 & & & \\ & 7.1 & & 0 \\ & & 64 & \\ & 0 & & 1 \\ & & & & 1 \end{bmatrix}$$

parameter corrections :

iter	Δx	Δy
1	-.536	-2.496
2	.0069	-.0007
3	-.0000018	-.0000003
4	.0000000005	.0000000007

final parameters :

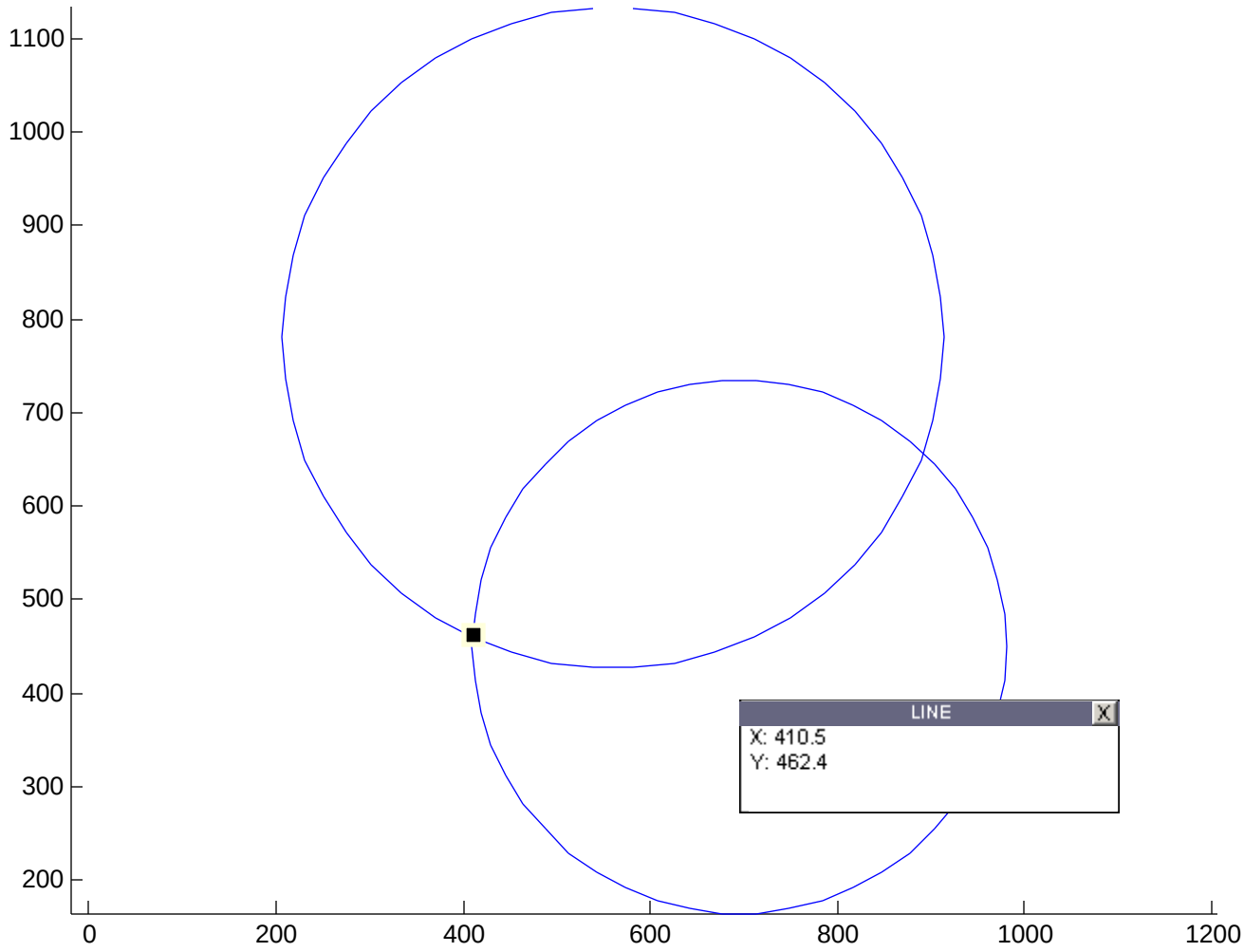
$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} 409.97 \\ 459.90 \end{bmatrix}$$

residuals :

$$V = \begin{bmatrix} .062 \\ -.049 \\ .1008 \\ -.128 \\ .088 \end{bmatrix}$$

adjusted obs :

$$\hat{l} = \begin{bmatrix} 353.51 \\ 285.20 \\ 270.56 \\ 343.05 \\ 298.37 \end{bmatrix}$$


1100
1000
900
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0
200
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800
1000
1200
LINE
X: 410.5
Y: 462.4

```
% dist5.m  13-oct-08
% initial approx (see graphics for approx)
x0=410.5;
y0=462.4;
n=5;
n0=2;
r=n-n0;
c=n;
u=n0;
B=zeros(c,u);
f=zeros(c,1);
sig0=0.08;
sig=[0.03;0.03;0.01;0.08;0.08];
w=zeros(n,1);
for i=1:n
  w(i)=sig0^2/sig(i)^2;
  end
W=diag(w);
X=[560;695;465;85;180];
Y=[780;450;195;350;650];
d=[353.45;285.25;270.55;343.18;298.28];
keep_going=1;
iter=1;
while (keep_going == 1)
  for i=1:n
    D=sqrt((x0-X(i))^2 + (y0-Y(i))^2);
    B(i,1)=-(x0-X(i))/D;
    B(i,2)=-(y0-Y(i))/D;
    Fi=d(i)-D;
    f(i)=-Fi;
    end
  N=B'*W*B;
  t=B'*W*f;
  if(iter==1)
    B
    f
    W
    end;
  iter
  del=inv(B'*W*B)*B'*W*f
  if (all(abs(del)<1.0e-06) | (iter > 10))
    keep_going=0;
    end
  x0=x0 + del(1);
  y0=y0 + del(2);
  iter=iter+1;
  end

[x0 y0]
v=f-B*del
dhat=d + v
```

```
dist5
B =
      0.42589      0.90477
      0.99905    -0.043544
      0.19971     -0.97986
     -0.94523      -0.3264
     -0.77559      0.63124
f =
      -2.4229
      -0.4799
       2.3474
       1.1803
      -1.0864
W =
       7.1111            0            0            0            0
            0       7.1111            0            0            0
            0            0           64            0            0
            0            0            0            1            0
            0            0            0            0            1
iter =
     1
del =
     -0.53606
      -2.4965
iter =
     2
del =
    0.0069202
  -0.00071653
iter =
     3
del =
 -1.8437e-006
 -3.7711e-007
iter =
     4
del =
  5.0073e-010
  7.4537e-011
ans =
       409.97        459.9
v =
     0.062305
     -0.04888
    0.0081451
     -0.12798
     0.088116
dhat =
       353.51
        285.2
       270.56
       343.05
       298.37
diary off
```