Adjustment of Geospatial Observations
EXAM 1 - 19 Oct 2009

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Name

→ 1 sheet of notes are allowed ←

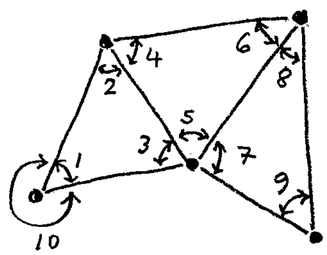
1. $\sigma_{x_1}^2 = 2$, $\sigma_{x_2}^2 = 2$, $\sigma_{x_1 x_2} = 1$

$Y_1 = X_1$

$Y_2 = X_1 + X_2$

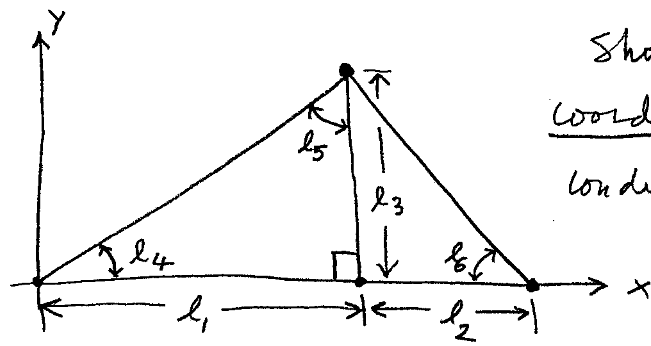
<u>Find</u> Σ_Y.

2.



10 angles are observed to determine the shape of the figure. (a) what are n_0 and r?
(b) write condition equations for the <u>observation only</u> method.

3.



Show n, n_0, and r. Select <u>point coordinates</u> as parameters and write condition equations for the <u>indirect observation</u> method.

4. The following condition equation is from a larger adjustment problem,

$$F = \ell_1 \cdot \sin(\ell_2) - \ell_3 / \sin(\ell_4)$$

Linearize this equation into the form

$$A v = f$$

Assume it is the first iteration. Show numerical values for A and f.

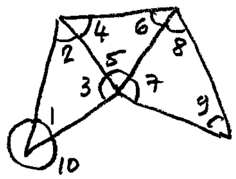
$$\vec{\ell} = \begin{bmatrix} \ell_1 \\ \ell_2 \\ \ell_3 \\ \ell_4 \end{bmatrix} = \begin{bmatrix} 10.0 \\ 30^\circ \\ 3.6 \\ 45^\circ \end{bmatrix}$$

(For partial derivatives assume angle units are <u>radians</u>)

Data 1 Exam 1 solution
22 Oct 2009

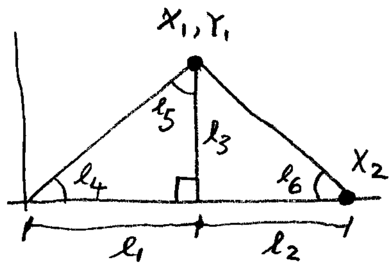
1. $\Sigma_x = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} \\ \sigma_{x_1x_2} & \sigma_{x_2}^2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, $\begin{matrix} Y_1 = X_1 \\ Y_2 = X_1 + X_2 \end{matrix}$, $\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$, $Y = AX$

$$\Sigma_Y = A\Sigma_x A^T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix}}}$$

2. 

$n = 10$ obs. only : $c = r$
$n_0 = 6$
$r = 4$

$\ell_1 + \ell_2 + \ell_3 = 180$
$\ell_4 + \ell_5 + \ell_6 = 180$
$\ell_7 + \ell_8 + 9_9 = 180$
$\ell_1 + \ell_{10} = 360$

3. 

$n = 6$ params: X_1, Y_1, X_2
$n_0 = 3$
$r = 3$

1. $\ell_1 = X_1$
2. $\ell_2 = X_2 - X_1$
3. $\ell_3 = Y_1$
4. $\ell_4 = \tan^{-1}(Y_1/X_1)$
5. $\ell_5 = \tan^{-1}(X_1/Y_1)$
6. $\ell_6 = \tan^{-1}\left(\dfrac{Y_1}{(X_2 - X_1)}\right)$

4. $\boxed{F = \ell_1 \sin(\ell_2) - \dfrac{\ell_3}{\sin(\ell_4)}}$ $\Rightarrow Av = f$ 1^{st} iteration, $\ell = \begin{bmatrix} 10.0 \\ 30° \\ 3.6 \\ 45° \end{bmatrix}$

$\dfrac{\partial F}{\partial \ell_1} = \sin(\ell_2)$ $\quad \partial F/\partial \ell_3 = -1/\sin(\ell_4)$

$\dfrac{\partial F}{\partial \ell_2} = \ell_1 \cdot \cos(\ell_2)$ $\quad \partial F/\partial \ell_4 = -\ell_3(-1)\dfrac{1}{\sin^2(\ell_4)} \cdot \cos(\ell_4) = \dfrac{\ell_3 \cdot \cos(\ell_4)}{\sin^2(\ell_4)}$

$$Av = f: \begin{bmatrix} \frac{\partial F}{\partial \ell_1} & \frac{\partial F}{\partial \ell_2} & \frac{\partial F}{\partial \ell_3} & \frac{\partial F}{\partial \ell_4} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = -F - A(\ell - \ell^0) = \text{zero for } 1^{st} \text{ iteration}$$

$$\begin{bmatrix} \sin(\ell_2) & \ell_1 \cdot \cos(\ell_2) & -1/\sin(\ell_4) & \dfrac{\ell_3 \cdot \cos(\ell_4)}{\sin^2(\ell_4)} \end{bmatrix}\begin{bmatrix} V \end{bmatrix} = -\left(\ell_1 \sin(\ell_2) - \ell_3/\sin(\ell_4)\right)$$

$$\begin{bmatrix} \sin(30°) & 10 \cdot \cos(30°) & \dfrac{-1}{\sin(45°)} & \dfrac{3.6 \cdot \cos(45°)}{\sin^2(45°)} \end{bmatrix}\begin{bmatrix} V \end{bmatrix} = -\left(10 \cdot \sin(30°) - 3.6/\sin(45°)\right)$$

$$\begin{bmatrix} 0.5 & 8.660 & -1.414 & 5.091 \end{bmatrix}\begin{bmatrix} V \end{bmatrix} = -(5 - 5.091) = .091$$