



Conclusion from histograms: "averaging" reduces spread or variability, averaging over more samples makes greater reduction in spread / variability. Standard deviation or variance is a measure of this spread / variability. See quantitative argument next page. (All of this assumes that all samples are independent and unbiased.) Averaging over more samples = greater redundancy in LS language.

```

hw21
meanx =
-7.3298e-005
meany =
-7.3298e-005
meanz =
-7.3298e-005
stdx =
0.9862
stdy =
0.4904
stdz =
0.2443
dsx =
0.7659
dsy =
0.3524
dsz =
0.1390
Ax =
7.6586e+003
Ay =
880.9563
Az =
86.8560
gx =
Columns 1 through 8
0.0054 0.0353 0.1442 0.3106 0.3894 0.2837 0.1094 0.0245
Columns 9 through 10
0.0031 0.0001
gy =
Columns 1 through 8
0.0079 0.0409 0.1635 0.5040 0.7912 0.7083 0.4370 0.1362
Columns 9 through 10
0.0454 0.0034
gz =
Columns 1 through 8
0.0230 0.1957 0.4605 0.9556 1.4161 1.6119 1.2895 0.7829
Columns 9 through 10
0.2993 0.1612
diary off

```

$\sigma_x = 1$
 $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}, n = 4 = \frac{\sigma_x}{2}$
 $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{16}} = \frac{\sigma_x}{4}$

```

% hw21.m

x=random('norm', 0, 1, 10000, 1);
y=zeros(2500, 1);
z=zeros(625, 1);

m=0;
for i=1:2500
    sm=0;
    for k=1:4
        sm=sm + x(m+k);
    end;
    y(i)=sm/4;
    m=m+4;
end

m=0;
for i=1:625
    sm=0;
    for k=1:16
        sm=sm + x(m+k);
    end;
    z(i)=sm/16;
    m=m+16;
end

meanx=mean(x)
meany=mean(y)
meanz=mean(z)
stdx=std(x)
stdy=std(y)
stdz=std(z)

[hx, sx]=hist(x);
[hy, sy]=hist(y);
[hz, sz]=hist(z);
dsx=sx(2)-sx(1)
dsy=sy(2)-sy(1)
dsz=sz(2)-sz(1)
Ax=0;
Ay=0;
Az=0;
for i=1:10
    Ax=Ax + hx(i)*dsx;
    Ay=Ay + hy(i)*dsy;
    Az=Az + hz(i)*dsz;
end
Ax
Ay
Az
gx=hx/Ax
gy=hy/Ay
gz=hz/Az

subplot(3, 1, 1);
bar(sx, gx);
ylabel('X');
axis([-4 4 0 1.5]);

subplot(3, 1, 2);
bar(sy, gy);
ylabel('Y');
axis([-4 4 0 1.5]);

subplot(3, 1, 3);
bar(sz, gz);
ylabel('Z');
axis([-4 4 0 1.5]);

```

hw22

```

B =
  -1  -1  -1  -1
  -1  -2  -1  -2
  -1  -3  -1  -3
  -1  -1  -2  -2
  -1  -2  -2  -4
  -1  -3  -2  -6
  -1  -1  -3  -3
  -1  -2  -3  -6
  -1  -3  -3  -9

```

```

f =
-1.5900
-1.7400
-2.3600
-1.7900
-2.4000
-2.9400
-2.1900
-2.8100
-3.4300

```

```

W =
Columns 1 through 8
  1.0000    0    0    0    0    0    0    0
    0    1.0000    0    0    0    0    0    0
    0    0    1.0000    0    0    0    0    0
    0    0    0    1.0000    0    0    0    0
    0    0    0    0    100.0000    0    0    0
    0    0    0    0    0    1.0000    0    0
    0    0    0    0    0    0    1.0000    0
    0    0    0    0    0    0    0    1.0000
    0    0    0    0    0    0    0    0

```

```

Column 9
  0
  0
  0
  0
  0
  0
  0
  0
  0
  0

```

```

1.0000
a0 = 0.9001
a1 = 0.2917
a2 = 0.2217
a3 = 0.1175

```

```

v =
-0.0591
 0.2001
-0.0107
 0.0801
-0.0032
-0.0166
 0.0193
 0.0434
 0.0676

```

```

zhat =
 1.5309
 1.9401
 2.3493
 1.8701
 2.3968
 2.9234
 2.2093
 2.8534
 3.4976

```

di ary off

```
% hw22. m 29-sep-09
```

```
n=9;
n0=4;
r=5;
u=4;
z=[1.59; 1.74; 2.36; 1.79; 2.40; 2.94; 2.19; 2.81; 3.43];
s=[0.1; 0.1; 0.1; 0.1; 0.1; 0.01; 0.1; 0.1; 0.1];
x=[1; 2; 3; 1; 2; 3; 1; 2; 3];
y=[1; 1; 1; 2; 2; 3; 3; 3];
B=zeros(n,u);
f=zeros(n,1);
s0=0.1;
W=zeros(n,n);
for i=1:n
    W(i,i)=s0^2 / s(i)^2;
end
for i=1:n
    B(i,:)=[-1 -x(i) -y(i) -x(i)*y(i)];
    f(i)=-z(i);
end
B
f
W

del = inv(B' * W * B) * B' * W * f;
a0=del(1)
a1=del(2)
a2=del(3)
a3=del(4)
v=f - B*del
zhat=z+v
```

hw23

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$f = \begin{bmatrix} 0.4300 \\ -0.3500 \\ -0.1100 \\ -0.0400 \end{bmatrix}$$

$$W = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$k = \begin{bmatrix} 0.6162 \\ -0.4850 \\ -0.4050 \\ -0.1275 \end{bmatrix}$$

$$v = \begin{bmatrix} 0.1312 \\ 0.0037 \\ 0.0837 \\ 0.2112 \\ 0.4850 \\ 0.4050 \\ 0.1275 \end{bmatrix}$$

$$\hat{l} = \begin{bmatrix} 30.0012 \\ 144.8038 \\ 50.3937 \\ 44.8012 \\ 174.8050 \\ 95.1950 \\ 195.1975 \end{bmatrix}$$

diary off

```
% hw23. m 29-sep-09
```

```
n=7;  
n0=3;  
r=4;  
c=r;  
l=[29.87; 144.80; 50.31; 44.59; 174.32; 94.79; 195.07];  
s=0.5;  
s0=0.5;  
W=eye(7);  
Q=W;  
A=[1 1 1 1 0 0 0;  
    1 1 0 0 -1 0 0;  
    0 0 1 1 0 -1 0;  
    0 1 1 0 0 0 -1];  
d=[270; 0; 0; 0];  
f=d - A*l;
```

```
A  
f  
W
```

```
Qe=A*Q*A';  
We=inv(Qe);  
k=We*f  
v=Q*A'*k  
lhat=l+v
```