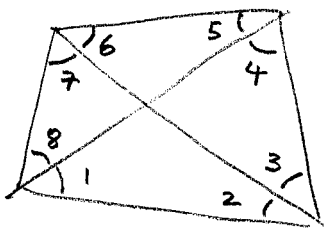
Data 1, 2009 HW3 braced quadrilateral
by observations only — 15 Oct 09



$n = 8$ w/ obs. only $c = r = 4$

$n_0 = 4$

$r = 4$

observations equal precision + uncorrelated.

$W = I_8$

1. $\hat{l}_1 + \hat{l}_2 + \hat{l}_3 + \hat{l}_4 = 180°$, $\hat{l}_1 + \hat{l}_2 + \hat{l}_3 + \hat{l}_4 - 180 = 0$
2. $\hat{l}_5 + \hat{l}_6 + \hat{l}_7 + \hat{l}_8 = 180°$, $\hat{l}_5 + \hat{l}_6 + \hat{l}_7 + \hat{l}_8 - 180 = 0$
3. $\hat{l}_1 + \hat{l}_2 + \hat{l}_7 + \hat{l}_8 = 180°$, $\hat{l}_1 + \hat{l}_2 + \hat{l}_7 + \hat{l}_8 - 180 = 0$
4. $\frac{\sin \hat{l}_5}{\sin \hat{l}_8} \cdot \frac{\sin \hat{l}_3}{\sin \hat{l}_6} \cdot \frac{\sin \hat{l}_7}{\sin \hat{l}_2} \cdot \frac{\sin \hat{l}_1}{\sin \hat{l}_4} - 1 = 0$

put in form $Ax = f$

coefficients of first 3 rows of A by inspection.

find partial derivatives of eqn. 4 for 2 cases: numerator & denominator

$$\frac{\partial F}{\partial l_5} = \cos l_5 \cdot \frac{1}{\cos l_8} \cdot \frac{\sin l_3}{\sin l_6} \cdot \frac{\sin l_7}{\sin l_2} \cdot \frac{\sin l_1}{\sin l_4} \qquad \text{(numerator)}$$

$$\frac{\partial F}{\partial l_8} = \frac{-\cos l_8}{\sin^2 l_8} \cdot \frac{\sin l_5}{1} \cdot \frac{\sin l_3}{\sin l_6} \cdot \frac{\sin l_7}{\sin l_2} \cdot \frac{\sin l_1}{\sin l_4}$$

Same form for all partial derivatives, ...

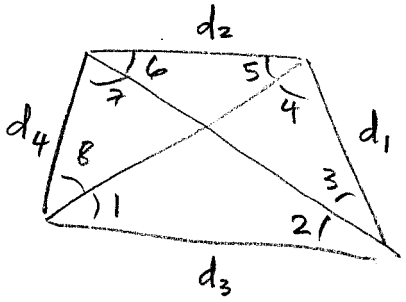
(for numbers see matlab listing)

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ \frac{\partial F_4}{\partial l_1} & \frac{\partial F_4}{\partial l_2} & \frac{\partial F_4}{\partial l_3} & \frac{\partial F_4}{\partial l_4} & \frac{\partial F_4}{\partial l_5} & \frac{\partial F_4}{\partial l_6} & \frac{\partial F_4}{\partial l_7} & \frac{\partial F_4}{\partial l_8} \end{bmatrix}$$

evaluate each iteration @ current value of observation, l°

$$f = -F - A(l - l^\circ) = \begin{bmatrix} -(l_1^\circ + l_2^\circ + l_3^\circ + l_4^\circ - \pi) \\ -(l_5^\circ + l_6^\circ + l_7^\circ + l_8^\circ - \pi) \\ -(l_1^\circ + l_2^\circ + l_7^\circ + l_8^\circ - \pi) \\ -F_4(l_1^\circ, \ldots, l_8^\circ) \end{bmatrix} - A(l - l^\circ)$$

evaluate F each iteration @ current value of observation, l°; "l" is original value.



development of 4th condition equation using law of sines : ($\sin\theta_i = s_i$) ←

$$\left.\begin{aligned} \frac{d_1}{s_6} = \frac{d_2}{s_3}, \quad d_2 = \frac{s_3}{s_6} d_1 \\ \frac{d_2}{s_8} = \frac{d_4}{s_5}, \quad d_4 = \frac{s_5}{s_8} d_2 \end{aligned}\right\} \quad d_4 = \frac{s_5}{s_8} \cdot \frac{s_3}{s_6} \cdot d_1$$

$$\left.\begin{aligned} \frac{d_1}{s_1} = \frac{d_3}{s_4}, \quad d_3 = \frac{s_4}{s_1} d_1 \\ \frac{d_3}{s_7} = \frac{d_4}{s_2}, \quad d_4 = \frac{s_2}{s_7} d_3 \end{aligned}\right\} \quad d_4 = \frac{s_2}{s_7} \cdot \frac{s_4}{s_1} \cdot d_1$$

$$\frac{s_5}{s_8} \cdot \frac{s_3}{s_6} \cdot d_1 = \frac{s_2}{s_7} \cdot \frac{s_4}{s_1} \cdot d_1$$

$$\frac{s_5}{s_8} \cdot \frac{s_3}{s_6} = \frac{s_2}{s_7} \cdot \frac{s_4}{s_1}$$

$$\frac{s_5}{s_8} \cdot \frac{s_3}{s_6} \cdot \frac{s_7}{s_2} \cdot \frac{s_1}{s_4} = 1$$

condition equation : $F = \frac{s_5}{s_8} \cdot \frac{s_3}{s_6} \cdot \frac{s_7}{s_2} \cdot \frac{s_1}{s_4} - 1 = 0$

$$F = \frac{\sin\theta_5}{\sin\theta_8} \cdot \frac{\sin\theta_3}{\sin\theta_6} \cdot \frac{\sin\theta_7}{\sin\theta_2} \cdot \frac{\sin\theta_1}{\sin\theta_4} - 1 = 0$$

note : Q_{vv} (obs. only) $= Q A^T W_e A Q$

```
brq_oo
niter =
     1
A =
  Columns 1 through 7
           1           1           1           1           0           0           0
           0           0           0           0           1           1           1
           1           1           0           0           0           0           1
       1.079      -1.6208     0.99999    -0.56583      1.7675    -0.99996     0.48282
  Column 8
           0
           1
           1
     -1.1396
f =
           0
  1.4544e-005
  6.7874e-005
 -4.2422e-005
We =
      0.38243     0.13391    -0.26632    -0.027201
      0.13391      0.3857    -0.26959    -0.032638
     -0.26632    -0.26959     0.53586     0.059763
    -0.027201    -0.032638    0.059763      0.09959
k =
 -1.4975e-005
 -1.1304e-005
  2.9915e-005
 -6.4319e-007
v =
  1.4246e-005
  1.5983e-005
 -1.5618e-005
 -1.4611e-005
  -1.244e-005
  -1.066e-005
  1.8301e-005
  1.9344e-005
phi =
  1.8933e-009
niter =
     2
phi =
  1.8933e-009
v =
  1.4246e-005
  1.5983e-005
 -1.5618e-005
 -1.4611e-005
 -1.2441e-005
  -1.066e-005
  1.8301e-005
  1.9344e-005
vs =
       2.9385
       3.2967
      -3.2215
      -3.0137
       -2.566
      -2.1988
       3.7748
       3.9901
Qvv =
  Columns 1 through 7
      0.57185     0.19385     0.22678   0.0075292    0.076597     -0.3109     0.23073
      0.19385     0.54171    0.031339      0.2331    -0.31052     0.04608    0.027698
      0.22678    0.031339     0.42762     0.31426     0.22923    0.028884   -0.070334
    0.0075292      0.2331     0.31426      0.4451   0.0046954     0.23594     -0.1881
     0.076597    -0.31052     0.22923   0.0046954     0.58146     0.18461     0.23328
      -0.3109     0.04608    0.028884     0.23594     0.18461     0.55056    0.025147
      0.23073    0.027698   -0.070334     -0.1881     0.23328    0.025147     0.43179
    0.0035727     0.23675    -0.18778   -0.052537  0.00064342     0.23967     0.30978
  Column 8
    0.0035727
      0.23675
     -0.18778
    -0.052537
   0.00064342
      0.23967
      0.30978
       0.4499
diary off
```

```
% brq_oo.m  15-oct-09
% braced quad obs only
%    sin 5   sin 3   sin 7   sin 1
% F= ----- * ----- * ----- * ----- - 1
%    sin 8   sin 6   sin 2   sin 4

n=8;
n0=4;
r=4;
c=4;
degrad=180/pi;
l=zeros(8,1);
l(1)=(42 + 49/60 + 32/3600)/degrad;
l(2)=(31 + 40/60 + 28/3600)/degrad;
l(3)=(45 + 00/60 + 05/3600)/degrad;
l(4)=(60 + 29/60 + 55/3600)/degrad;
l(5)=(29 + 30/60 + 03/3600)/degrad;
l(6)=(45 + 00/60 + 08/3600)/degrad;
l(7)=(64 + 13/60 + 44/3600)/degrad;
l(8)=(41 + 16/60 + 02/3600)/degrad;

l0=l;
old_l0=0;
W=eye(8);
Q=eye(8);
keep_going=1;
niter=0;

while((keep_going==1) && (niter < 10))
  niter=niter+1
  A=zeros(c,8);
  f=zeros(c,1);
  F=zeros(c,1);
  A(1,:)=[1 1 1 1 0 0 0 0];
  A(2,:)=[0 0 0 0 1 1 1 1];
  A(3,:)=[1 1 0 0 0 0 1 1];
  s1=sin(l0(1));
  s2=sin(l0(2));
  s3=sin(l0(3));
  s4=sin(l0(4));
  s5=sin(l0(5));
  s6=sin(l0(6));
  s7=sin(l0(7));
  s8=sin(l0(8));
  A(4,1)=(s5/s8)*(s3/s6)*(s7/s2)*( 1/s4)*cos(l0(1));
  A(4,2)=(s5/s8)*(s3/s6)*(s7/1 )*(s1/s4)*(-1/s2^2)*cos(l0(2));
  A(4,3)=(s5/s8)*( 1/s6)*(s7/s2)*(s1/s4)*cos(l0(3));
  A(4,4)=(s5/s8)*(s3/s6)*(s7/s2)*(s1/1 )*(-1/s4^2)*cos(l0(4));
  A(4,5)=( 1/s8)*(s3/s6)*(s7/s2)*(s1/s4)*cos(l0(5));
  A(4,6)=(s5/s8)*(s3/1 )*(s7/s2)*(s1/s4)*(-1/s6^2)*cos(l0(6));
  A(4,7)=(s5/s8)*(s3/s6)*( 1/s2)*(s1/s4)*cos(l0(7));
  A(4,8)=(s5/1 )*(s3/s6)*(s7/s2)*(s1/s4)*(-1/s8^2)*cos(l0(8));
  F(1)=l0(1)+l0(2)+l0(3)+l0(4)-pi;
  F(2)=l0(5)+l0(6)+l0(7)+l0(8)-pi;
  F(3)=l0(1)+l0(2)+l0(7)+l0(8)-pi;
  F(4)=(s5/s8)*(s3/s6)*(s7/s2)*(s1/s4) - 1;
  f= -F - A*(l-l0);
  Qe=A*Q*A';
  We=inv(Qe);
  k=We*f;
  v=Q*A'*k;
  if(niter == 1)
    A
    f
    We
    k
    v
    end
  lhat=l+v;
  l0=lhat;
  delta_l=l0 - old_l0;
  if(all(abs(delta_l) < 1.0e-08))
    keep_going=0;
    end
  phi=v'*W*v
  old_l0=l0;
  end
v
vs=v*degrad*60*60
% W=Q=I
Qvv=A'*We*A
```