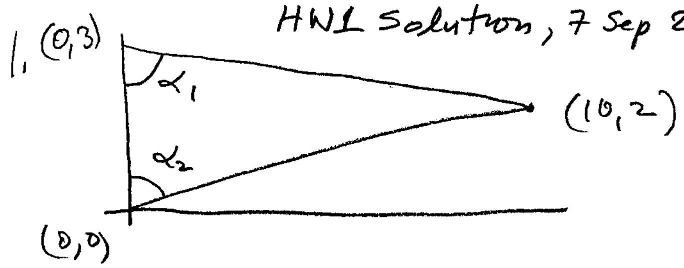
# HW1 Solution, 7 Sep 2011




$$\left.\begin{aligned}\alpha_1 &= 84.289406862^\circ \\ \alpha_2 &= 78.690067526^\circ\end{aligned}\right\}\text{ nominal angles}$$

$$\left.\begin{aligned}\alpha_1 &= \alpha_{1\,nom} + e_1 \\ \alpha_2 &= \alpha_{2\,nom} + e_2\end{aligned}\right\}\ e \sim N(0, 0.1)$$

slope for line @ $\alpha_1 = -\tan(90-\alpha_1) = m_1$

slope for line @ $\alpha_2 = \tan(90-\alpha_2) = m_2$

equation for line 1 : $\dfrac{y-3}{x-0} = m_1, \quad y = m_1 x + 3$

equation for line 2 : $\dfrac{y-0}{x-0} = m_2, \quad y = m_2 x$

set up equations to solve for $x, y$ : $m_1 x - y = -3$

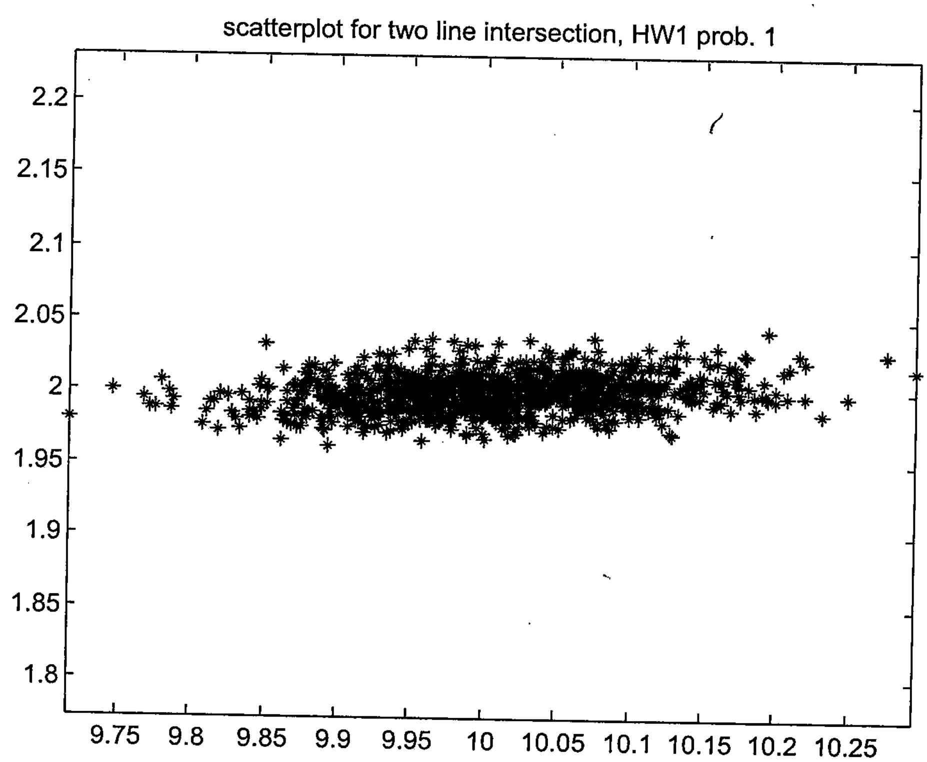
$m_2 x - y = 0$

$$\begin{bmatrix} m_1 & -1 \\ m_2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$ , plot $x, y$ , do this 1000 times

Matlab Code

```
% lin_intsc.m  7-sep-2011
% make scatterplot of line intersecti

degrad=180/pi;
alpha1=84.289406862;
alpha2=78.690067526;
for i=1:1000
  e=random('norm',0,0.1,2,1);
  a1=alpha1 + e(1);
  a2=alpha2 + e(2);
  m1=-tan((90-a1)/degrad);
  m2=tan((90-a2)/degrad);
  A=[m1 -1;m2 -1];
  b=[-3;0];
  x=inv(A)*b;
  px=x(1);
  py=x(2);
  plot(px,py,'*');
  if(i == 1)
    hold on
    end
  end
axis equal
title('scatterplot for two line intersection, HW1 prob. 1');
```




Weak intersection geometry in $x$-direction causes the scatterplot to be extended in $x$, compared to $y$.

2. Parabola Fit

$n = 4$

$n_0 = 3$ just count parameters in given equation.

$\underline{x\text{'s constant}}$

$r = 1$

Use indirect observations, choose $n_0 = 3$ parameters : $a_0, a_1, a_2$
Write $n$ condition equations expressing each observation in terms of the parameters : $\hat{y}_i = a_0 + a_1 x_i + a_2 x_i^2$

$$\hat{y}_1 = 1.52 + V_1 = a_0 + a_1 \cdot 1 + a_2 \cdot 1, \quad V_1 = a_0 + a_1 + a_2 - 1.52$$
$$\hat{y}_2 = 1.05 + V_2 = a_0 + a_1 \cdot 2 + a_2 \cdot 4, \quad V_2 = a_0 + 2a_1 + 4a_2 - 1.05$$
$$\hat{y}_3 = 1.46 + V_3 = a_0 + a_1 \cdot 3 + a_2 \cdot 9, \quad V_3 = a_0 + 3a_1 + 9a_2 - 1.46 \qquad (*)$$
$$\hat{y}_4 = 3.11 + V_4 = a_0 + a_1 \cdot 4 + a_2 \cdot 16, \quad V_4 = a_0 + 4a_1 + 16a_2 - 3.11$$

plug into $\Phi = V_1^2 + V_2^2 + V_3^2 + V_4^2$ (equal weights, $w_i = 1$)

$$\Phi = (a_0 + a_1 + a_2 - 1.52)^2 + (a_0 + 2a_1 + 4a_2 - 1.05)^2 + (a_0 + 3a_1 + 9a_2 - 1.46)^2 + (a_0 + 4a_1 + 16a_2 - 3.11)^2$$

differentiate $\Phi$ with respect to $a_0, a_1, a_2$ and set = zero

$$\frac{\partial \Phi}{\partial a_0} = \cancel{2}(a_0 + a_1 + a_2 - 1.52) + \cancel{2}(a_0 + 2a_1 + 4a_2 - 1.05) + \cancel{2}(a_0 + 3a_1 + 9a_2 - 1.46) + \cancel{2}(a_0 + 4a_1 + 16a_2 - 3.11) = 0$$

chain rule

$$\frac{\partial \Phi}{\partial a_1} = \cancel{2}(a_0 + a_1 + a_2 - 1.52) + \cancel{2}(a_0 + 2a_1 + 4a_2 - 1.05)(2) + \cancel{2}(a_0 + 3a_1 + 9a_2 - 1.46)(3) + \cancel{2}(a_0 + 4a_1 + 16a_2 - 3.11)(4) = 0$$

$$\frac{\partial \Phi}{\partial a_2} = \cancel{2}(a_0 + a_1 + a_2 - 1.52) + \cancel{2}(a_0 + 2a_1 + 4a_2 - 1.05)(4) + \cancel{2}(a_0 + 3a_1 + 9a_2 - 1.46)(9) + \cancel{2}(a_0 + 4a_1 + 16a_2 - 3.11)(16) = 0$$

cancel common factor of "2".

Collect terms and simplify,

$$4a_0 + 10a_1 + 30a_2 = 7.14$$
$$10a_0 + 30a_1 + 100a_2 = 20.44$$
$$30a_0 + 100a_1 + 354a_2 = 68.62$$

$$\begin{bmatrix} 4 & 10 & 30 \\ 10 & 30 & 100 \\ 30 & 100 & 354 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 7.14 \\ 20.44 \\ 68.62 \end{bmatrix}$$

(Solve this in Matlab)

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3.140 \\ -2.132 \\ 0.530 \end{bmatrix}$$

go back and plug into the condition equations (*) in order to obtain the residuals, $v$.

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} = \begin{bmatrix} 0.018 \\ -0.054 \\ 0.054 \\ -0.018 \end{bmatrix}, \quad \begin{aligned} \hat{l}_1 &= l_1 + v_1 = 1.538 \\ \hat{l}_2 &= l_2 + v_2 = 0.9960 \\ \hat{l}_3 &= l_3 + v_3 = 1.514 \\ \hat{l}_4 &= l_4 + v_4 = 3.092 \end{aligned}$$