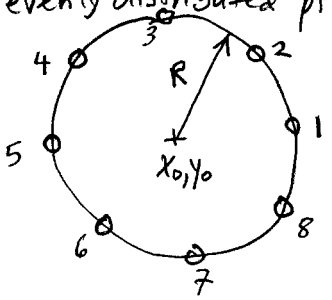
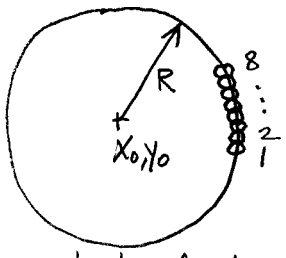
HW5 Circle Fit — Solution

evenly distributed pts.





clustered pts.

$n = 16 \quad (8 \times 2)$

$n_0 = 3 + 8 = 11$

$r = 5$

$u = 3$

$c = 8 \quad (r + u)$

General Least Squares

$\sigma_x = \sigma_y = 0.04$

choose $\sigma_0 = 0.04$, $\Rightarrow W = I_{16}$

$$F_i = R - \left[(x_i - x_0)^2 + (y_i - y_0)^2 \right]^{1/2} = 0, \quad \cdots D_i$$

$$\frac{\partial F}{\partial x_i} = -\frac{1}{2}[\cdot]^{-1/2} \cdot 2 \cdot (x_i - x_0) = -\frac{(x_i - x_0)}{D_i}$$

$$\frac{\partial F}{\partial y_i} = \frac{-(y_i - y_0)}{D_i}$$

$$\frac{\partial F}{\partial x_0} = +(x_i - x_0)/D_i$$

$$\frac{\partial F}{\partial y_0} = +(y_i - y_0)/D_i$$

$$\frac{\partial F}{\partial R} = 1$$

$$f_i = -F_i(\ell_i^0, x^0) - A_i(\ell - \ell^0)$$

---

evenly distributed pts.

X_0 499.994 273

Y_0 1999.970 276

R 99.962 878

	V_x	V_y	$\hat{x}$	$\hat{y}$
1.	.010151	−.000000	599.957	1999.960
2.	−.006389	−.006411	570.557	2070.777
3.	−.000006	−.028847	⋮	⋮
4.	−.038925	.038935		
5.	.036396	−.000006		
6.	−.007155	−.007152		
7.	.000003	.009404		
8.	.005924	−.005919		

$$\chi^{2*} = \frac{V^T W V}{\sigma_0^2} = 3.521, \quad CV_1 = 0.831, \quad CV_2 = 12.832, \quad \alpha = 0.05$$

PASS

assume pass $\Sigma = \sigma_0^2 Q$

confidence interval for R, $P = 99\%$

$$\hat{R} \pm Z_{\frac{P+1}{2}} \cdot \sigma_R$$

$$\hat{R} \pm Z_{.995} \cdot \sigma_R$$

$$99.962878 \pm 2.575829 \cdot 0.014142$$

$$\pm 0.036427$$

$99.926451 \longrightarrow 99.999305$

(width 0.072854)

$$\Sigma = \begin{matrix} \\ X_0 \\ Y_0 \\ R \end{matrix} \begin{bmatrix} .000400 & 0 & 0 \\ 0 & .000399 & 0 \\ 0 & 0 & .000200 \end{bmatrix}$$

$\lambda_1 = .0004$

$\lambda_2 = .0004$

Major axis $= \sqrt{\lambda_1 \cdot \chi^2_{2,.99}} = .0607$

minor axis $= \sqrt{\lambda_2 \cdot \chi^2_{2,.99}} = .0607$

($\chi^2_{2,.99} = 9.21$)

Confidence Circle Radius $= .0607$ (99%)

$\theta = -6.79^\circ$

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assume not pass $\Sigma = \hat{\sigma}_0^2 Q$

$$\hat{R} \pm t_{\frac{P+1}{2},5} \cdot \sigma_R$$

$$\hat{R} \pm t_{.995,5} \cdot \sigma_R$$

$$99.962878 \pm 4.032143 \cdot .0118679$$

$$\pm .047853$$

$99.915025 \longrightarrow 100.010731$

(width 0.095706)

$$\Sigma = \begin{matrix} \\ X_0 \\ Y_0 \\ R \end{matrix} \begin{bmatrix} .0002817 & 0 & 0 \\ 0 & .0002816 & 0 \\ 0 & 0 & .0001408 \end{bmatrix}$$

smaller than previous!

$\lambda_1 = .0002817$

$\lambda_2 = .0002816$

major axis $= \sqrt{\lambda_1 \cdot 2 \cdot F_{2,5,.99}} = .0865$

minor axis $= \sqrt{\lambda_2 \cdot 2 \cdot F_{2,5,.99}} = .0865$

($F_{2,5,.99} = 13.27393$)

$\theta = -6.79^\circ$

Cluster point distribution

X_0 497.926407
Y_0 2000.081036
R 102.033604

	V_x	V_y	$\hat{x}$	$\hat{Y}$
1.	-.020112	.005647	596.161	1972.498
2.	.029931	-.006255	597.802	1979.207
3.	.021072	-.002931	⋮	⋮
4.	-.034853	.002389		
5.	-.002006	.000001		
6.	-.021925	-.001496		
7.	.044459	.006088		
8.	-.016565	-.003442		

$\chi^{2*} = \frac{V^T W V}{\sigma_0^2} = 3.644$, $CV_1 = 0.831$, $CV_2 = 12.832$, $\alpha = 0.05$

$\Rightarrow$ PASS No indication via residuals that we have a poor determination of the circle.

Assume PASS

$$\Sigma = \sigma_0^2 Q = \begin{bmatrix} X_0 \\ Y_0 \\ R \end{bmatrix} \begin{bmatrix} 1.7600 & -.0606 & -1.7395 \\ -.0606 & .0103 & .0602 \\ -1.7395 & .0602 & 1.7194 \end{bmatrix}$$

$\hat{R} \pm z_{.995} \cdot \sigma_R$

$102.033604 \pm 2.575829 \cdot 1.311290$

± 3.377658

$98.655946 \rightarrow 105.411262$

(width 6.755)

$\lambda_1 = 1.762185$

$\lambda_2 = .008253$

$maj = \sqrt{\lambda_1 \cdot \chi^2_{2,.99}} = 4.028$

$min = \sqrt{\lambda_2 \cdot \chi^2_{2,.99}} = .275$

(9.21)

$\theta = -1.98^\circ$

99% circle radius = 3.4

Assume NOT PASS

$$\Sigma = \hat{\sigma}_0^2 Q = \begin{bmatrix} X_0 \\ Y_0 \\ R \end{bmatrix} \begin{bmatrix} 1.283011 & -.044211 & -1.268049 \\ -.044211 & .007546 & .043902 \\ -1.268049 & .043902 & 1.253414 \end{bmatrix}$$

$\hat{\sigma}_0^2 = .001166$

$\hat{R} \pm t_{.995} \cdot \sigma_R$

$102.033604 \pm \underbrace{4.032143 \cdot 1.119560}$

$\pm \quad 4.514226$

$97.519378 \rightarrow 106.547830$

(width: 9.028)

$\lambda_1 = 1.284542$

$\lambda_2 = .006016$

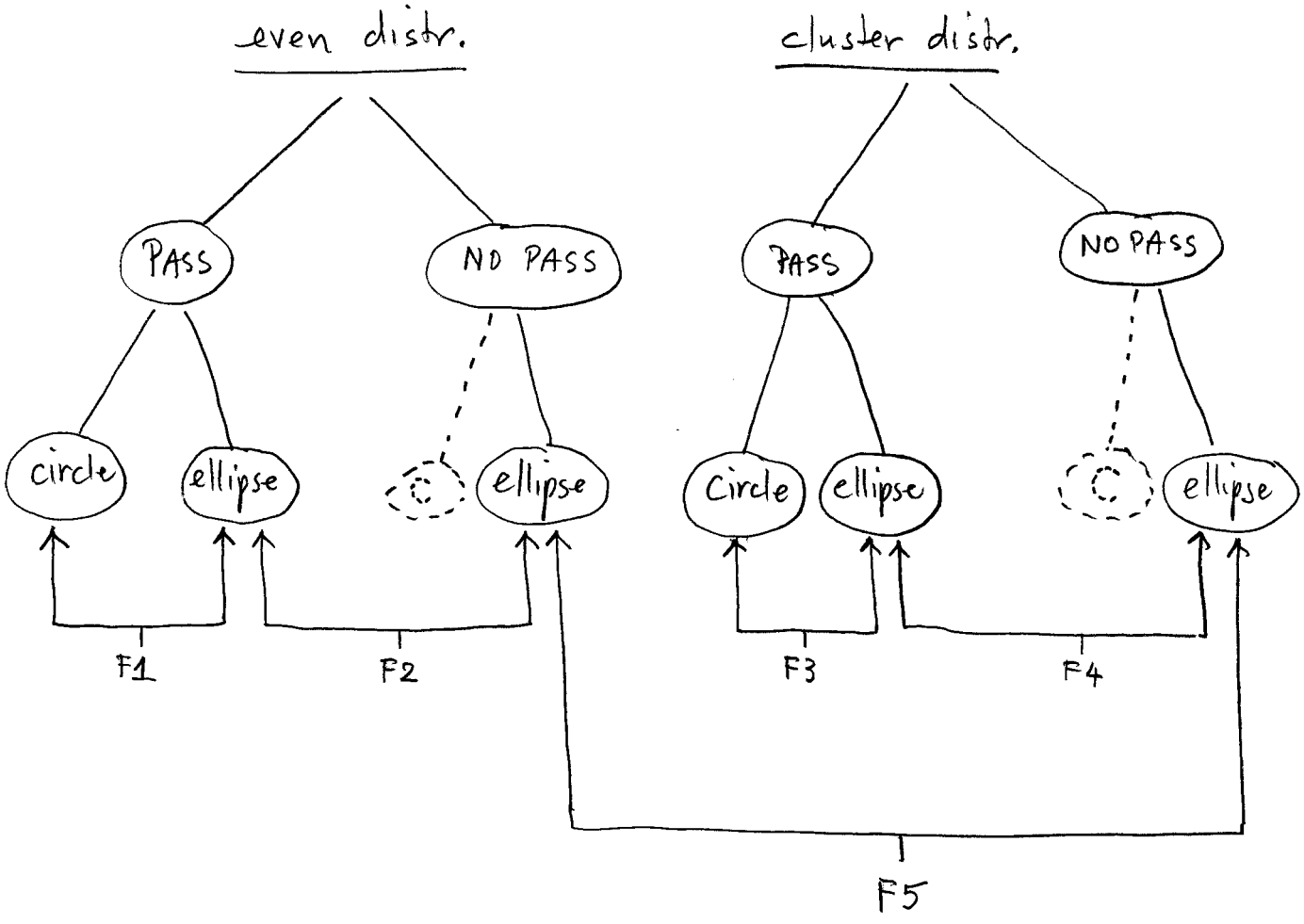
$maj = \sqrt{\lambda_1 \cdot 2 \cdot F_{2,5,.99}} = 5.84$

$min = \sqrt{\lambda_2 \cdot 2 \cdot F_{2,5,.99}} = .400$

13.3

$\theta = -1.98°$

Figures



	C.I.	C.R. Circle	C.R. Ellipse
PASS	$\frac{\hat{\bar{x}} - \mu_x}{\sigma} \sim z$	$(\vec{\bar{x}} - \vec{\mu}_x) \notin \Sigma \sim MVN$	$(\vec{\bar{x}} - \vec{\mu}_x)^T \Sigma (\vec{\bar{x}} - \vec{\mu}_x) \sim \chi^2_n$
NO PASS	$\frac{\hat{\bar{x}} - \mu_x}{\hat{\sigma}} \sim t_r$	$(\vec{\bar{x}} - \mu_x) \notin \hat{\Sigma} \sim$? (MVT?)	$(\vec{\bar{x}} - \vec{\mu}_x)^T \hat{\Sigma} (\vec{\bar{x}} - \vec{\mu}_x) \sim nF_{n,r}$

Fig. 1

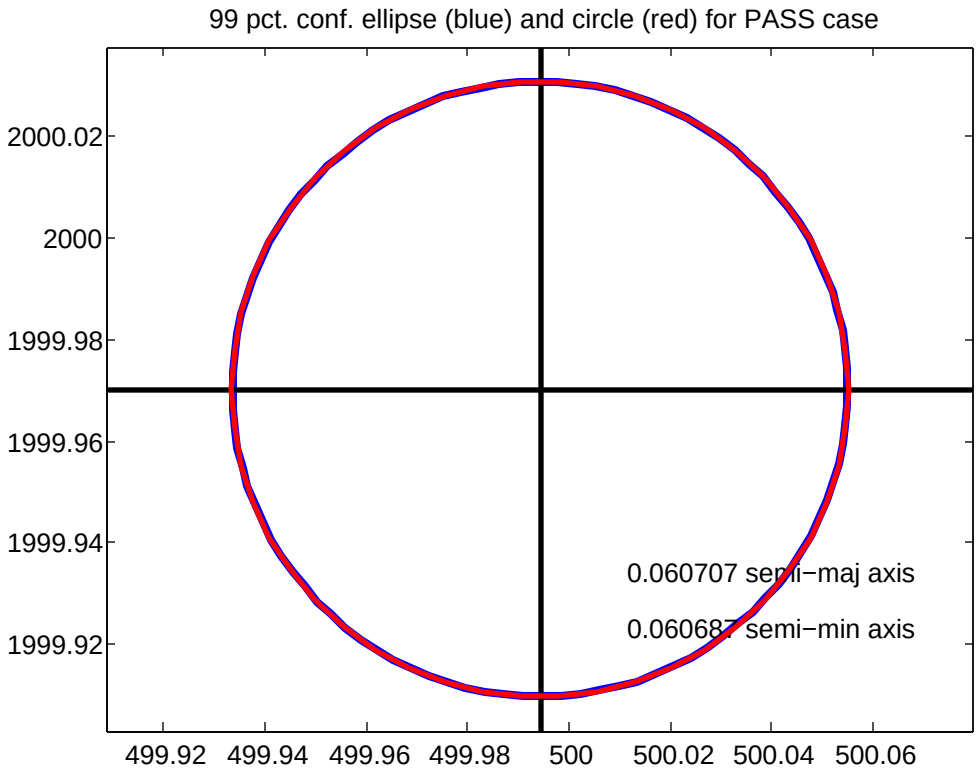

99 pct. conf. ellipse (blue) and circle (red) for PASS case
2000.02
2000
1999.98
1999.96
1999.94
1999.92
499.92
499.94
499.96
499.98
500
500.02
500.04
500.06
0.060707 semi−maj axis
0.060687 semi−min axis

Fig. 2

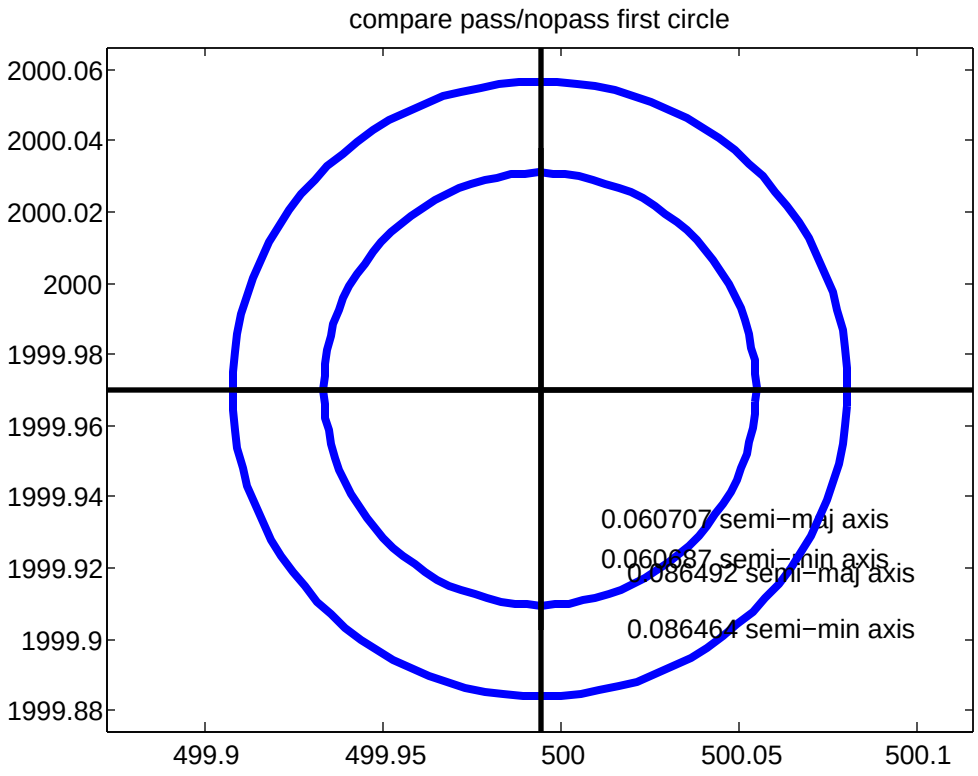

compare pass/nopass first circle
2000.06
2000.04
2000.02
2000
1999.98
1999.96
1999.94
1999.92
1999.9
1999.88
499.9
499.95
500
500.05
500.1
0.060707 semi−maj axis
0.060687 semi−min axis
0.086492 semi−maj axis
0.086464 semi−min axis

Fig. 3

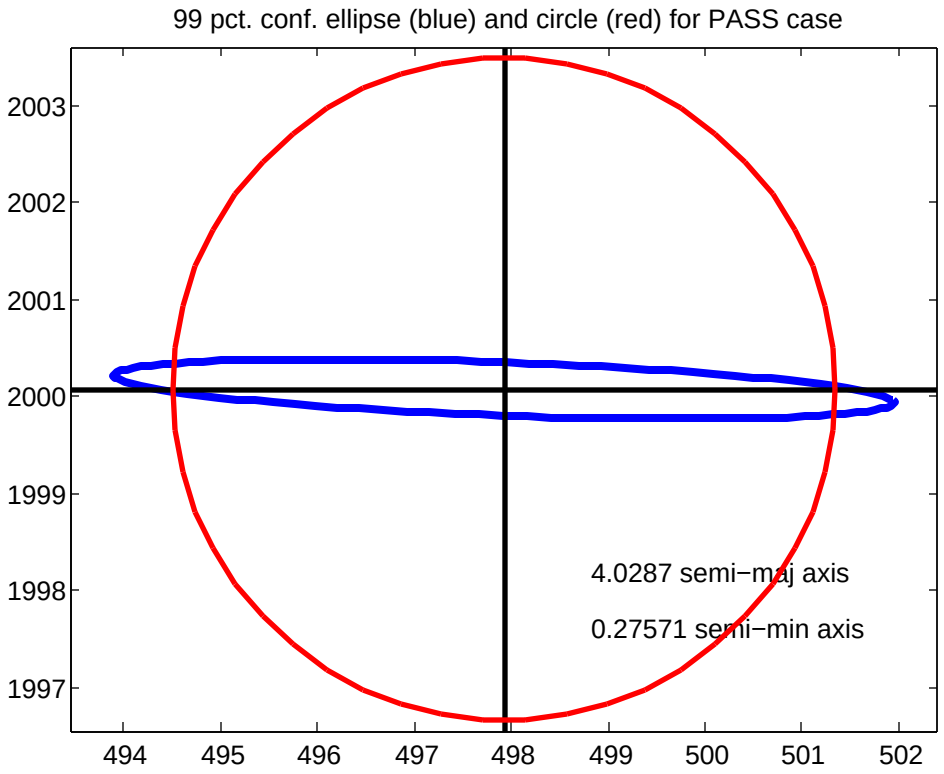

99 pct. conf. ellipse (blue) and circle (red) for PASS case
2003
2002
2001
2000
1999
1998
1997
494
495
496
497
498
499
500
501
502
4.0287 semi−maj axis
0.27571 semi−min axis

Fig. 4

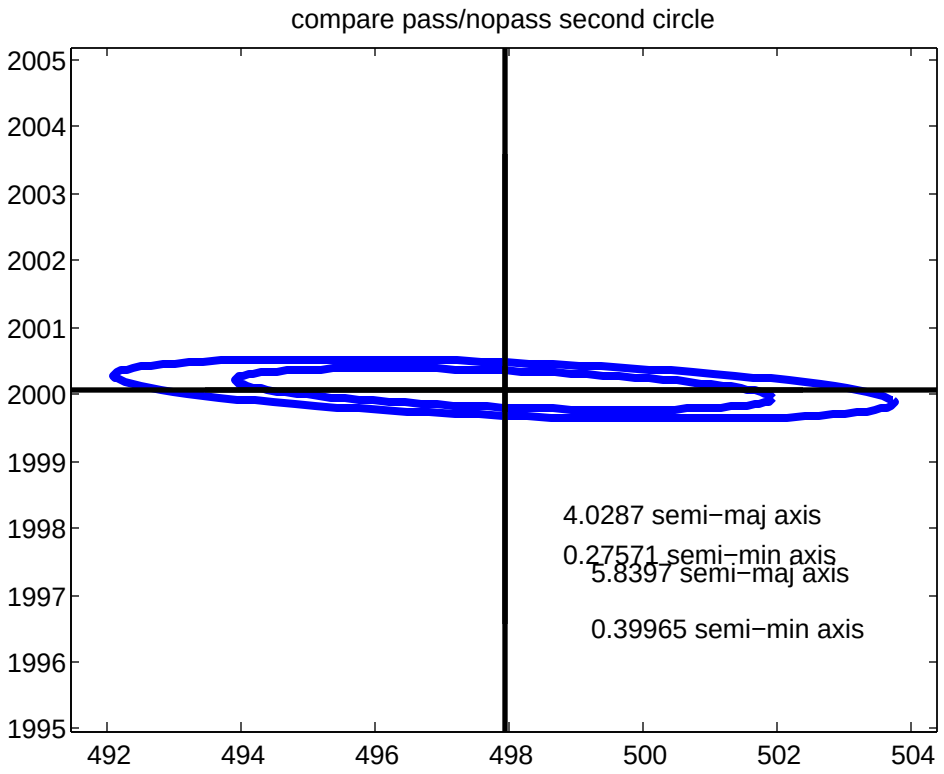
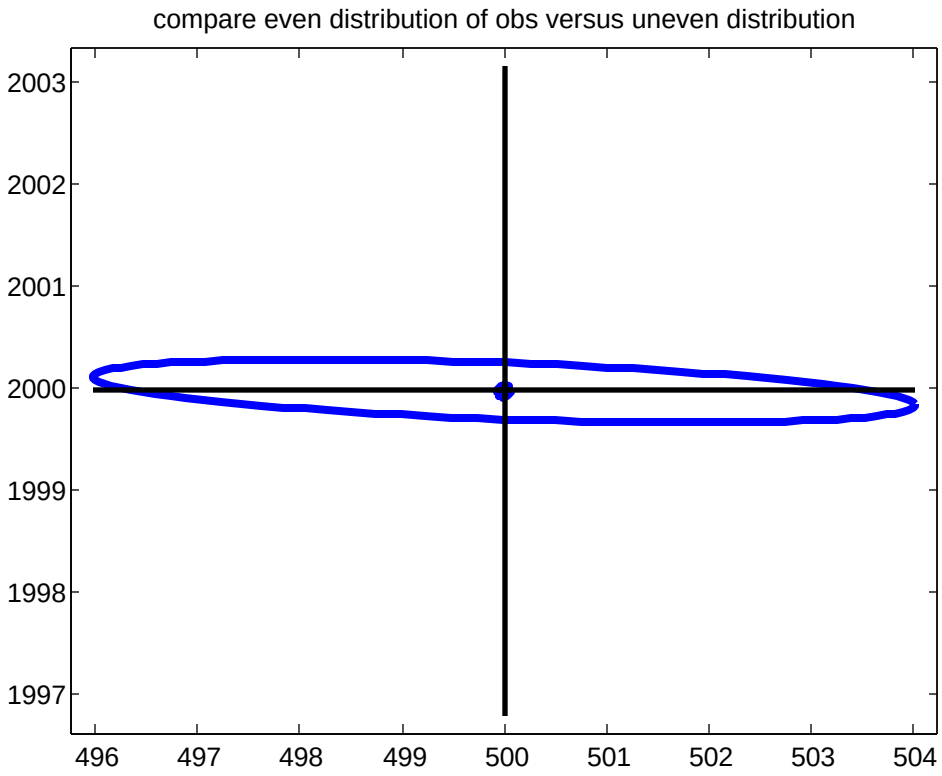

compare pass/nopass second circle
2005
2004
2003
2002
2001
2000
1999
1998
1997
1996
1995
492
494
496
498
500
502
504
4.0287 semi−maj axis
0.27571 semi−min axis
5.8397 semi−maj axis
0.39965 semi−min axis

Fig. 5


compare even distribution of obs versus uneven distribution
2003
2002
2001
2000
1999
1998
1997
496
497
498
499
500
501
502
503
504

```
cir2
iter =
     1
ans =
  Columns 1 through 2
         0.994396811289146         -1.02981437142362
  Column 3
         0.957753530867133
iter =
     2
ans =
  Columns 1 through 2
     -0.000123336682429769     9.04253089871899e-005
  Column 3
       0.00512540610245305
iter =
     3
ans =
  Columns 1 through 2
     3.54977182352335e-008    -5.10792939880728e-008
  Column 3
     3.21176946921674e-010
we have converged
show results
parameters X0 y0 R
ans =
  Columns 1 through 2
          499.994273510104          1999.97027600281
  Column 3
          99.9628789372908
residuals
v =
         0.0101519191107017
     -1.04371023242896e-006
       -0.00638989696721481
       -0.00641199058127963
     -6.26809524980091e-006
         -0.028847419619151
        -0.0389251234781494
         0.0389352908575883
         0.0363961429538113
     -6.45081769538078e-006
       -0.00715500290731551
       -0.00715257394548798
       3.5061726543031e-006
        0.00940401336073484
        0.00592472321076222
       -0.00591982554447667
ans =
  Columns 1 through 2
           3.5211977765451         0.831211613486658
  Column 3
            12.83250199403
we pass the global test
assume that we pass
sig0 =
```

0.04

```
Qxx =
  Columns 1 through 2
         0.250079597827706    -1.92190642788199e-005
    -1.92190642788199e-005         0.249920490695652
     3.47744770389023e-005    -5.61905258469171e-005
  Column 3
     3.47744770389023e-005
    -5.61905258469171e-005
         0.125000017467835
SIGxx =
  Columns 1 through 2
       0.000400127356524329    -3.07505028461119e-008
    -3.07505028461119e-008      0.000399872785113043
     5.56391632622436e-008    -8.99048413550674e-008
  Column 3
     5.56391632622436e-008
    -8.99048413550674e-008
      0.000200000027948535
lam1 =
      0.000400131018305437
lam2 =
      0.000399869123331935
theta =
         -0.11852224754071
sqrt_lam1 =
        0.0200032751894643
sqrt_lam2 =
        0.0199967278156186
C =
          3.03485425877029
confidence interval
z =
           2.5758293035489
sigR =
        0.0141421366118609
half_intvl =
         0.036427729899623
limits =
          99.9264512073911          99.9993066671904
maj_axis =
        0.0607070248980998
min_axis =
        0.0606871545727005
theta_deg =
         -6.79082456212217
n =
    50
assume that we do not pass
sig0_sqr_hat =
       0.00112678328849443
ans =
        0.0335675928313966
Qxx =
  Columns 1 through 2
         0.250079597827706    -1.92190642788199e-005
```

```
                                    c1
    -1.92190642788199e-005          0.249920490695652
     3.47744770389023e-005    -5.61905258469171e-005
  Column 3
     3.47744770389023e-005
    -5.61905258469171e-005
         0.125000017467835
SIGxx =
  Columns 1 through 2
      0.000281785511625667    -2.16557204498746e-008
    -2.16557204498746e-008      0.000281606232368189
     3.91832995935684e-008    -6.33145454960207e-008
  Column 3
     3.91832995935684e-008
    -6.33145454960207e-008
      0.000140847930744268
lam1 =
      0.000281788090396766
lam2 =
       0.00028160365359709
theta =
         -0.11852224754075
sqrt_lam1 =
        0.0167865449213579
sqrt_lam2 =
        0.0167810504318737
C =
          5.15246224867389
confidence interval
t =
          4.03214298355522
sigR =
        0.0118679370888233
half_intvl =
        0.0478532192619737
limits =
          99.9150257180288          100.010732156553
maj_axis =
         0.086492038992965
min_axis =
        0.0864637288433222
theta_deg =
         -6.79082456212448
diary off
```

```
cir2
iter =
     1
ans =
  Columns 1 through 2
        -1.07131817385743        -0.909499035092949
  Column 3
         3.02718925332738
iter =
     2
ans =
  Columns 1 through 2
     -0.00226649108714128      -0.00946158040793876
  Column 3
      0.00640703077908711
iter =
     3
ans =
  Columns 1 through 2
    -7.71273271053417e-006    -2.55030799326726e-006
  Column 3
     7.9616516964671e-006
iter =
     4
ans =
  Columns 1 through 2
     8.4333461883588e-008    -2.99650778879982e-008
  Column 3
    -8.42780367429456e-008
we have converged
show results
parameters X0 y0 R
ans =
  Columns 1 through 2
         497.926407706656          2000.08103680423
  Column 3
          102.03360416148
residuals
v =
        -0.020112557841077
       0.00564743169421792
        0.0299316653355414
      -0.00625581039604679
        0.0210728106837821
      -0.00293110899010636
       -0.0348536742000465
        0.0023890887122682
      -0.00200699042539831
     1.22023432460059e-006
       -0.0219258669930837
      -0.00149683756461987
        0.0444596734637943
       0.00608865076811573
       -0.0165650600235123
      -0.00344263445815343
ans =
```

```
                                    c2
  Columns 1 through 2
          3.64474292355896          0.831211613486658
  Column 3
           12.83250199403
we pass the global test
assume that we pass
sig0 =
                     0.04
Qxx =
  Columns 1 through 2
          1100.05338377417          -37.9067322907465
         -37.9067322907465           6.47068189354424
         -1087.22464167793           37.6418155840154
  Column 3
         -1087.22464167793
          37.6418155840154
          1074.67658391501
SIGxx =
  Columns 1 through 2
          1.76008541403868        -0.0606507716651944
       -0.0606507716651944         0.0103530910296708
         -1.73955942668469         0.0602269049344246
  Column 3
         -1.73955942668469
        0.0602269049344246
          1.71948253426401
lam1 =
          1.76218522487815
lam2 =
       0.00825328019020266
theta =
       -0.0346075147003763
sqrt_lam1 =
          1.32747324827213
sqrt_lam2 =
        0.0908475656812149
C =
          3.03485425877029
confidence interval
z =
           2.5758293035489
sigR =
          1.31129040805766
half_intvl =
           3.3776602585375
limits =
          98.6559439029426          105.411264420018
maj_axis =
          4.02868784092229
min_axis =
         0.275709121606549
theta_deg =
         -1.98286453166185
n =
    50
assume that we do not pass
```

```
sig0_sqr_hat =
       0.00116631773553887
ans =
        0.0341513943425282
Qxx =
  Columns 1 through 2
          1100.05338377417         -37.9067322907465
         -37.9067322907465          6.47068189354424
         -1087.22464167793          37.6418155840154
  Column 3
         -1087.22464167793
          37.6418155840154
          1074.67658391501
SIGxx =
  Columns 1 through 2
          1.28301177153536       -0.0442112941670215
       -0.0442112941670215       0.00754687105347086
         -1.26804938210386        0.0439023171135204
  Column 3
         -1.26804938210386
        0.0439023171135204
           1.2534143597884
lam1 =
          1.28454242567496
lam2 =
       0.00601621691387821
theta =
       -0.0346075147003763
sqrt_lam1 =
          1.13337655952245
sqrt_lam2 =
        0.0775642760159483
C =
          5.15246224867389
confidence interval
t =
          4.03214298355522
sigR =
          1.11955989557879
half_intvl =
          4.51422557762784
limits =
          97.5193785838523          106.547829739108
maj_axis =
          5.83967993647135
min_axis =
         0.399647004017896
theta_deg =
         -1.98286453166185
diary off
```

```
% cir2.m - 1-nov-2011 revised from cir.m ce506_04/hw7
% solve circle fit problem by gen ls
% select two circles by commenting/uncommenting the XY data below

% (a)
%X=[599.947;570.563;500.016;429.358;399.995;429.305;499.957;570.702];
%Y=[1999.960;2070.783;2099.962;2070.625;1999.988;1929.305;1899.998;192
9.321];
% (b)
X=[596.181;597.772;598.966;599.756;599.962;599.745;598.972;597.842];
Y=[1972.492;1979.213;1986.027;1993.101;2000.019;2007.032;2013.919;2020
.846];
sig=0.04; % actually it is 0.05
X0=499;
Y0=2001;
R=99.0;

format compact
format long g
[npt,dum]=size(X);
n=2*npt; % 16
n0=3 + npt; % 3 + 8 = 11
r=n-n0; % 16 - 11 = 5
u=3;
c=r+u; % 8

sig0=sig;
sig0_sqr=sig0*sig0;

A=zeros(c,n);
B=zeros(c,3);
f=zeros(c,1);
W=eye(n)*((sig0_sqr)/(sig*sig));

% obs order x1,y1,x2,y2, ...
% param order X0,Y0,R
% collect the observations into l & l0 vectors

l=zeros(n,1);
l0=zeros(n,1);
v=zeros(n,1);
for i=1:npt
  idx2=2*i;
  idx1=idx2-1;
  l(idx1)=X(i);
  l(idx2)=Y(i);
  end
l0=l;

Q=inv(W);
phi=9.99e+8;
thresh=1.0e-06;
converged=0;
imax=10;
iter=1;
```

```
while((iter <= imax) & (converged == 0))
  iter
  for i=1:c
    compR=sqrt((X(i)-X0)^2 + (Y(i)-Y0)^2);
    colidx=(i-1)*2 + 1;
    A(i,colidx:colidx+1)=[-(X(i)-X0)/compR  -(Y(i)-Y0)/compR];
    B(i,:)=[(X(i)-X0)/compR  (Y(i)-Y0)/compR  1];
    F=R - compR;
    f(i)= -F;
    end
  f=f - A*(l-l0);
  Qe=A*Q*A';
  We=inv(Qe);
  N=B'*We*B;
  t=B'*We*f;
  del=inv(N)*t;
  del'
  % cn=cond(N)
  X0=X0 + del(1);
  Y0=Y0 + del(2);
  R=R + del(3);
  v=Q*A'*We*(f-B*del);
  l0=l+v;
  for i=1:npt
    idx2=2*i;
    idx1=idx2-1;
    X(i)=l0(idx1);
    Y(i)=l0(idx2);
    end
  new_phi=v'*W*v;
  if(abs(new_phi - phi)/phi < thresh)
    disp('we have converged');
    converged=1;
    phi=new_phi;
    end
  phi=new_phi;
  iter=iter + 1;
  end

% ok now show results if converged
if(converged == 1)
  disp('show results');
  disp('parameters X0 y0 R');
  [X0 Y0 R]
  disp('residuals');
  v
  Ni=inv(N);
  Qxx=Ni;

  cov=Ni(1:2,1:2);
  sig0_sqr_hat=phi/r;
  test_stat=phi/(sig0^2);
  alph=0.05; % level of significance
  parg=alph/2;
  cv1=icdf('chi2',parg,r);
  parg=1 - alph/2;
```

```
  cv2=icdf('chi2',parg,r);
  [test_stat cv1 cv2]

  if((test_stat > cv1) & (test_stat < cv2))
    pass_test=1;
    disp('we pass the global test');
  else
    pass_test=0;
    disp('we failed the global test');
    end

  % ok ignore global test and do error prop under both
  % assumptions
  P=0.99;
  disp('assume that we pass');
  figure(1);
  SIGxx=sig0^2*Qxx;
  cov2=SIGxx(1:2,1:2);
  sig0
  Qxx
  SIGxx
  [lam1,lam2,theta]=eigval(cov2);
  lam1
  lam2
  theta
  sqrt_lam1=sqrt(lam1)
  sqrt_lam2=sqrt(lam2)
  C=sqrt(icdf('chi2',P,2))

  % 99 pct. conf. interval for R
  disp('confidence interval');
  z=icdf('norm',(P+1)/2,0,1)
  sigR=sqrt(SIGxx(3,3))
  half_intvl=z*sigR

  limits=[R-half_intvl R+half_intvl]

  % 99 pct. conf. region for X0,Y0
  maj_axis=C*sqrt(lam1);
  min_axis=C*sqrt(lam2);
  plot_ell(X0,Y0,maj_axis,min_axis,theta);
  maj_axis
  min_axis
  theta_deg=theta*57.29577951;
  theta_deg

  % 99 pct. conf. circle for X0,Y0
  radius=cep2(P,cov2);
  draw_cir(X0,Y0,radius);
  title('99 pct. conf. ellipse (blue) and circle (red) for PASS
case');
  axis equal
  % done figure 1

  disp('assume that we do not pass');
  figure(2);
```

```
  SIGxx=sig0_sqr_hat*Qxx;
  cov2=SIGxx(1:2,1:2);
  sig0_sqr_hat
  sqrt(sig0_sqr_hat)
  Qxx
  SIGxx
  [lam1,lam2,theta]=eigval(cov2);
  lam1
  lam2
  theta
  sqrt_lam1=sqrt(lam1)
  sqrt_lam2=sqrt(lam2)
  C=sqrt(2*icdf('f',P,2,r))

  % 99 pct. conf. interval for R
  disp('confidence interval');
  t=icdf('t',(P+1)/2,r)
  sigR=sqrt(SIGxx(3,3))
  half_intvl=t*sigR
  limits=[R-half_intvl R+half_intvl]

  % 99 pct. conf. region for X0,xy0
  maj_axis=C*sqrt(lam1);
  min_axis=C*sqrt(lam2);
  plot_ell(X0,Y0,maj_axis,min_axis,theta);
  maj_axis
  min_axis
  theta_deg=theta*57.29577951;
  theta_deg
  title('99 pct. conf. ellipse for DO NOT PASS case');
  axis equal
  % done figure 2

else
  disp('we did not converge');
  %
  %
  end
```

```
% cep2.m 11-nov-04
% for given 2x2 covariance and probability P,
% compute radius yielding P under bivariate normal
% syntax res=cep2(P,cov);
% original in d:\classes\ce603_03\

function res=cep2(P,cov)
sx2=cov(1,1);
sy2=cov(2,2);
sxy=cov(1,2);
sx=sqrt(sx2);
sy=sqrt(sy2);
long=max([sx sy]);
dr=long/100;
t1=2*pi*sqrt(det(cov));
term1=1/t1;
covi=inv(cov);
X=zeros(2,1);
degrad=180/(pi);
nth=300;
dth=pi/nth;
accumP=0;
rr=0;
while(accumP < 0.5*P)
  rp=rr + 0.5*dr;
  tt=0;
  for j=1:nth
    thp=tt + 0.5*dth;
    X(1)=rp*cos(thp);
    X(2)=rp*sin(thp);
    term2=-0.5*(X'*covi*X);
    f=term1*exp(term2);
    dens=f;
    %mu=[0 0];
    %XX=[X(1) X(2)];
    %dens=mvnpdf(XX,mu,cov);
    da=rp*dth*dr;
    accumP=accumP + da*dens;
    tt=tt + dth;
    end
  rr=rr + dr;
  end
res=rr;
```

```
% draw_cir.m  13-oct-08
function result=draw_cir(x0,y0,r)
xi=x0+r;
yi=y0;
n=50
degrad=180/pi;
dth=2*pi/n;
rth=0;
for i=1:n
  rth=rth+dth;
  costh=cos(rth);
  sinth=sin(rth);
  xip1=x0 + r*costh;
  yip1=y0 + r*sinth;
  plot([xi xip1],[yi yip1],'b-','linewidth',1);
  if(i==1)
    hold on
    end
  xi=xip1;
  yi=yip1;
  end
result=0;
```

```
% draw_ell.m  22-oct-08
% function to draw ellipse

function result=draw_ell(xorg,yorg,a,b,theta)

th=theta;
x0=a;
y0=0;
nseg=50;
dalpha=2*pi/nseg;
for i=1:nseg
  alpha=i*dalpha;
  x1=a*cos(alpha);
  y1=b*sin(alpha);
  px0=xorg + cos(th)*x0 - sin(th)*y0;
  py0=yorg + sin(th)*x0 + cos(th)*y0;
  px1=xorg + cos(th)*x1 - sin(th)*y1;
  py1=yorg + sin(th)*x1 + cos(th)*y1;
  plot([px0 px1],[py0 py1],'r-','linewidth',2);
  if(i == 1)
    hold on
    end
  x0=x1;
  y0=y1;
  end
result=0;
```

```
% plot_ell.m  9-dec-04
% plot ellipse given major axis, minor axis
% and ccw angle from x, to major axis

function plot_ell(x0,y0,maj_ax,min_ax,theta)
a=maj_ax;
b=min_ax;
px=zeros(101,1);
py=zeros(101,1);
for i=1:100
  alph=(i/100)*2*pi;
  xx=a*cos(alph);
  yy=b*sin(alph);
  px(i)= cos(-theta)*xx + sin(-theta)*yy;
  py(i)=-sin(-theta)*xx + cos(-theta)*yy;
  end

px(101)=px(1);
py(101)=py(1);
px=px + x0;
py=py + y0;

plot(px,py,'linewidth',3);
axis equal
hold on
scale_ax(0.9);
% title('confidence ellipse');

v=axis;
px=[x0; x0];
py=[v(3); v(4)];
plot(px,py,'linewidth',2,'color','black');
px=[v(1); v(2)];
py=[y0; y0];
plot(px,py,'linewidth',2,'color','black');

y_range=v(4)-v(3);
x_range=v(2)-v(1);
lin_spc=y_range/12;
locx=v(1) + 0.6*x_range;
locy=v(3) + 0.4*y_range;

str=[num2str(maj_ax) ' semi-maj axis'];
text(locx,locy-2*lin_spc,str);
str=[num2str(min_ax) ' semi-min axis'];
text(locx,locy-3*lin_spc,str);
```