4-1

$\vec{a}, \vec{b}$ linearly dependent if
$k_1\vec{a} + k_2\vec{b} = 0$
otherwise linearly independent

$[1\ 2\ 3]$
$[2\ 4\ 6]$ $\Rightarrow$ dependent

$$\left.\begin{matrix}[1\ 2\ 3] \\ [0\ 2\ 0] \\ [1\ 4\ 3]\end{matrix}\right\} \text{dependent}$$

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4-2

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \text{(square)}$$

rank: number of linearly independent rows or columns
"full rank" all r/c are independent
if not fullrank $\Rightarrow$ rank deficient

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4-3

indirect observations

all columns of parameter coefficient matrix must be independent

observation only

all equations must be independent

matrix of coefficients must have full row rank

n

m rank $<= n$

n

m rank $<= m$

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4-4

observation only method

constrained minimization

substitution method

minimize x^2+y^2

subject to $y = 0.2x + 5$

$$y^2 = 0.04x^2 + 25 + \underbrace{2 \cdot 0.2x \cdot 5}_{2x}$$

$$\Phi = x^2 + 0.04x^2 + 25 + 2x$$

$$= 1.04x^2 + 2x + 25$$

$$\frac{\partial \Phi}{\partial x} = 2.08x + 2 = 0, \quad 2.08x = -2$$

$$y = 4.8077, \quad x = -0.9615$$

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Method of Langrange Mult

4-5

$$2x - 0.2\lambda = 0$$
$$2y + \lambda = 0$$
$$-0.2x + y = 5$$

$$\Phi' = x^2 + y^2 + \lambda(y - 0.2x - 5)$$

$$\frac{\partial \Phi'}{\partial x} = 2x - 0.2\lambda = 0$$

$$\frac{\partial \Phi'}{\partial y} = 2y + \lambda = 0$$

$$\frac{\partial \Phi'}{\partial \lambda} = y - 0.2x - 5 = 0$$

$$\begin{bmatrix} 2 & 0 & -0.2 \\ 0 & 2 & 1 \\ -0.2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ \lambda \end{bmatrix} = \begin{bmatrix} 0.9615 \\ 4.8077 \\ -9.6154 \end{bmatrix}$$

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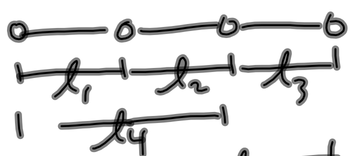
for LS : $\Phi' = V_1^2 + V_2^2 + \cdots + V_n^2 + \lambda_1(-) + \lambda_2(-) + \cdots$ 4-6



$n = 5$
$n_0 = 3$
$r = 2$

$\hat{\ell}_4 = \hat{\ell}_1 + \hat{\ell}_2$
$\hat{\ell}_5 = \hat{\ell}_2 + \ell_3$

$$\ell = \begin{bmatrix} 20 \\ 18 \\ 21 \\ 37.6 \\ 39.2 \end{bmatrix}$$

equal precision uncorrelated

$\ell_1 = 20$
$\ell_2 = 18$
$\ell_3 = 21$
$\ell_4 = 37.6$
$\ell_5 = 39.2$

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$\hat{l}_4 = \hat{l}_1 + \hat{l}_2$ Lag. Mult. 4-7

$\hat{l}_5 = \hat{l}_2 + l_3$

$l_4 + v_4 = l_1 + v_1 + l_2 + v_2$

$l_5 + v_5 = l_2 + v_2 + l_3 + v_3$

$v_4 - v_1 - v_2 = -l_4 + l_1 + l_2 = 0{,}4$

$v_5 - v_2 - v_3 = \underline{\underline{-l_5 + l_2 + l_3}} = -0{,}2$

$$\boxed{\begin{aligned} v_4 - v_1 - v_2 &= 0{,}4 \\ v_5 - v_2 - v_3 &= -0{,}2 \end{aligned}}$$

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$$\phi' = v_1^2 + v_2^2 + v_3^2 + v_4^2 + v_5^2 + \underline{\underline{2}}\lambda_1 (v_4 - v_1 - v_2 - 0{,}4) + \underline{\underline{2}}\lambda_2 (v_5 - v_2 - v_3 + 0{,}2)$$ 4-8

$$\frac{\partial \phi'}{\partial v_1} = 2v_1 - 2\lambda_1 = 0$$

$$\frac{\partial \phi'}{\partial v_2} = 2v_2 - 2\lambda_1 - 2\lambda_2 = 0$$

$$\frac{\partial \phi'}{\partial v_3} = 2v_3 - 2\lambda_2 = 0$$

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