20-1

confidence interval

- Selecting P
- solve for $F(z)$
- Solve for z
- get σ_x, $\hat{x}$
- construct interval

} assumed passed Global test

if not pass global test

OR

if no test to make, then

$$\underline{\underline{\hat{\Sigma} = \hat{\sigma}_0^2 Q}}, \quad \underline{\underline{\hat{\sigma}_0^2 = \frac{v^T W v}{r}}}$$

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$$\frac{\hat{x} - \mu_x}{\hat{\sigma}_x} \sim t_r \quad (\text{NOT } z)$$

$\hat{\Sigma}$ not Σ

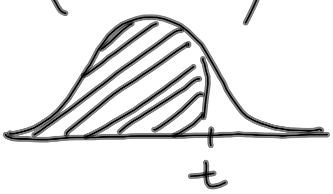
$$\text{Prob}\left(-t < \frac{\hat{x} - \mu_x}{\hat{\sigma}_x} < +t\right) = P$$

$$t = \text{icdf}\left('t', r, \frac{P+1}{2}\right)$$

$$\text{Prob}\left(\hat{x} - t\hat{\sigma}_x < \mu_x < \hat{x} + t\hat{\sigma}_x\right) = P$$

$$F(t) = \tfrac{1}{2} + \tfrac{P}{2} = \frac{P+1}{2}$$

$$t = F^{-1}(F(t))$$



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$\hat{x} \pm t\hat{\sigma}_x$

$\hat{x}-t\hat{\sigma}_x \qquad \hat{x} \qquad \hat{x}+t\hat{\sigma}_x$

$$\frac{v^T W v}{\sigma_0^2} \sim \chi^2_r$$

Theorem: $\vec{x} \sim N(0, \Sigma)$
if $A \cdot \Sigma$: idempotent + rank r

$$\left.\begin{matrix} \vec{x} \sim N(0,\Sigma) \\ \text{if } A\cdot\Sigma : \text{idempotent + rank } r \end{matrix}\right\} x^T A x \sim \chi^2_r$$

$$B^2 = B$$

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$$\left.\begin{matrix} v \sim N(0, \Sigma_{vv}) \\ A : \Sigma_{\ell\ell}^{-1}, \quad \underline{\Sigma_{\ell\ell}^{-1}}\Sigma_{vv} : \text{idempotent + rank } r \end{matrix}\right\} v^T \Sigma_{\ell\ell}^{-1} v \sim \chi^2_r$$

$$\underline{\Sigma_{\ell\ell}^{-1} = \frac{1}{\sigma_0^2} W}, \quad \underline{\Sigma_{vv} = \sigma_0^2 Q_{vv}}, \quad \underline{\Sigma_{\ell\ell}^{-1}\Sigma_{vv}} = W Q_{vv}$$

$$(WQ_{vv})^2 = \left[W\left(Q - BN^{-1}B^T\right)\right]^2 = \left(\underline{\underline{I - WBN^{-1}B^T}}\right)^2$$

$$(I - WBN^{-1}B^T)(I - WBN^{-1}B^T)$$

$$I + WBN^{-1}\underbrace{B^T W B}\underbrace{N}^{-1}B^T - WBN^{-1}B^T - WBN^{-1}B^T$$

$$I + WBN^{-1}B^T - 2WBN^{-1}B^T$$

$$I - WBN^{-1}B^T$$

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$v^T \Sigma_{ee}^{-1} v \sim \chi^2_r$ to justify the global test on reference variance 20-5

$\Sigma_{ee}^{-1} = \frac{1}{\sigma_0^2} W$

$$\boxed{\frac{v^T W v}{\sigma_0^2} \sim \chi^2_r}$$

Constructing confidence Region (2D)

$y = \underline{\underline{(\vec{x}-\vec{\mu}_x)^T \Sigma_{xx}^{-1} (\vec{x}-\vec{\mu}_x)}} \sim \chi^2_n$ n: dim. of vector

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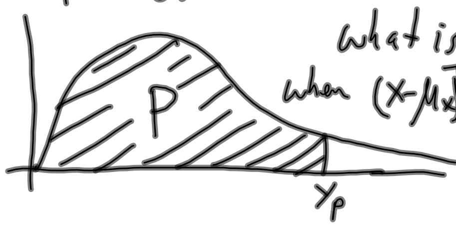
$(\vec{x}-\vec{\mu}_x)^T \Sigma_{xx}^{-1} (\vec{x}-\vec{\mu}_x)$ $\begin{bmatrix} 1/\sigma_1^2 & & \\ & \frac{1}{\sigma_2^2} & \\ & & \frac{1}{\sigma_3^2} \end{bmatrix}$ 20-6

$\frac{(x_1-\mu_{x_1})^2}{\sigma_1^2} + \frac{(x_2-\mu_{x_2})^2}{\sigma_2^2} + \frac{(x_3-\mu_{x_3})^2}{\sigma_3^2}$

$z_1^2 + z_2^2 + z_3^2 \sim \chi^2_3$

$z_1^2 + z_2^2 \sim \chi^2_2$



what is locus of x_1, x_2 when $(x-\mu_x)^T \Sigma_{xx}^{-1} (x-\mu_x) < y_p$

$\vec{x}: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

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eigenvalues

$Av = \lambda v$

$Av - \lambda v = 0$

$Av - I\lambda v = 0$

$(A - I\lambda)v = 0$

if system has nontrivial solution, then matrix singular

$\underbrace{\det(A - \lambda I) = 0}$

characteristic equation

$\boxed{\text{assume } 2\times 2}$

$Av_1 = \lambda_1 v_1$

$Av_2 = \lambda_2 v_2$

$$A\begin{bmatrix} v_1 v_2 \end{bmatrix} = \begin{bmatrix} v_1 v_2 \end{bmatrix}\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$AV = V\Lambda$$

A symmetric ⇒
λ real
V's orthogonal

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$$AV = V\Lambda$$

$$A = V\Lambda V^{-1}$$

eigenvalue decomposition

$(\vec{x} - \vec{\mu}_x)^T \Sigma_{xx}^{-1} (x - \mu_x) < y_p$

$\Sigma = V\Lambda V^{-1}$

$\Sigma = R^T D R$

$R \Sigma R^T = D$

$D: \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}, \quad D^{-1} \begin{bmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{bmatrix}$

$D^{-1} = R \Sigma^{-1} R^T$

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$\underline{\underline{R^T R = I}}$, $\vec{w} = R(\vec{x}-\vec{\mu}_x)$ 20-9

$w^T = [R(\vec{x}-\vec{\mu}_x)]^T = (\vec{x}-\vec{\mu}_x)^T R^T$ ✓

$$\text{Prob}\left[\underbrace{(x-\mu_x)^T R^T}\underbrace{R\,\Sigma_{xx}^{-1} R^T}\underbrace{R\,(x-\mu_x)} < \chi^2_{P,2}\right] = P$$

$$\text{Prob}\left[w^T D^{-1} w < \chi^2_{P,2}\right] = P$$

$$\text{Prob}\left[\begin{bmatrix} w_1 & w_2\end{bmatrix}\begin{bmatrix} 1/d_1 & 0 \\ 0 & \frac{1}{d_2}\end{bmatrix}\begin{pmatrix} w_1 \\ w_2\end{pmatrix} < \chi^2_{P,2}\right] = P$$

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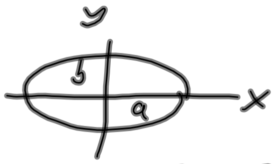
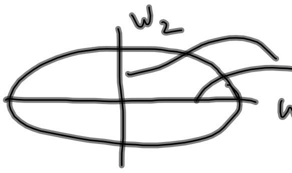
$$\text{Prob}\left[\frac{w_1^2}{d_1} + \frac{w_2^2}{d_2} < \chi^2_{P,2}\right] = P$$

$$\text{Prob}\left[\frac{w_1^2}{d_1\chi^2_{P,2}} + \frac{w_2^2}{d_2\chi^2_{P,2}} < 1\right] = P$$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$:



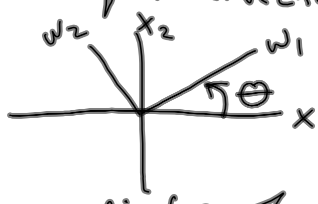


semi major $\sqrt{\lambda_1 \chi^2_{P,2}}$

semi minor $\sqrt{\lambda_2 \chi^2_{P,2}}$

$W = R(\vec{x}-\vec{\mu})$

rows of R : e.vectors



w_1, w_2 in directions of eigenvectors

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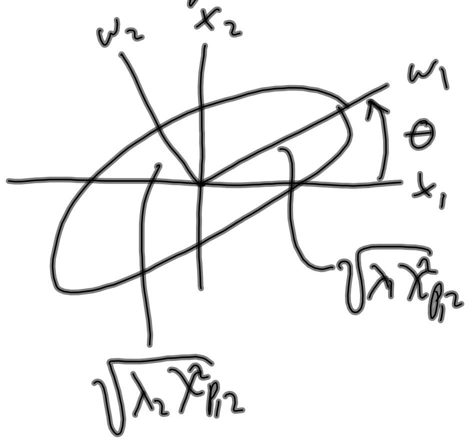
conf. ellipse: assumes pass global test! 20-11



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