General LS: $Av + B\Delta = f = -F(\ell^0, x^0) - A(\ell - \ell^0)$ 22-1

$\phi' = v^T W v - 2\overline{k^T(Av + B\Delta - f)}$, take deriv. w.r.t. v, Δ, k

$$\left.\begin{aligned} v^TW - k^TA &= 0 \\ -k^TB &= 0 \\ -(Av + B\Delta - f)^T &= 0 \end{aligned}\right\} \;\; \left.\begin{aligned} Wv - A^Tk &= 0 \\ -B^Tk &= 0 \\ -(Av + B\Delta - f) &= 0 \end{aligned}\right\} \;\; \left.\begin{aligned} -Wv + A^Tk &= 0 \\ Av + B\Delta &= f \\ B^Tk &= 0 \end{aligned}\right\} \; v, k, \Delta$$

transp, $-(\times -1)$

$$\begin{bmatrix} -W & A^T & 0 \\ A & 0 & B \\ 0 & B^T & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix}$$

full normal equations for GLS

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$$\left.\begin{aligned} -Wv + A^Tk &= 0 \\ Av + B\Delta &= f \\ B^Tk &= 0 \end{aligned}\right\}$$

$Wv = A^Tk$

$v = QA^Tk$ *

$\boxed{\text{forward elimination}}$

$AQA^Tk + B\Delta = f$

$AQA^Tk = f - B\Delta$

$k = (AQA^T)^{-1}(f - B\Delta)$ *

$B^T W_e (f - B\Delta) = 0$

$-B^T W_e B\Delta + B^T W_e f = 0$

FE, BS

$B^T W_e B\Delta = B^T W_e f$ $\rightarrow$ I/o $B^T W B\Delta = B^T W f$

$\Delta = (B^T W_e B)^{-1} B^T W_e f$

$k = W_e(f - B\Delta)$

$v = QA^Tk$

$x^0_{new} = x^0_{old} + \Delta$

$\ell^0_{new} = \ell_{original} + v$

$\delta\ell = \ell^0_{new} - \ell^0_{previous}$

$AQA^T = Q_e$

$\ell_e = A\ell$

$W_e = Q_e^{-1}$

22-2

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22-3

W, Q
$n \times n$

$l_e = A\,l$
$(c,1) \quad (c,n)(n,1)$

W_e, Q_e
$c \times c$

$W,\ A: \frac{\partial F}{\partial l},\ B: \frac{\partial F}{\partial X}$

$f = -F(l^0, x^0) - A(l - l^0)$

if you choose to linearize always at original observations then ...

$\rightarrow f - F(l, x^0)$

still need to iterate to refine parameters

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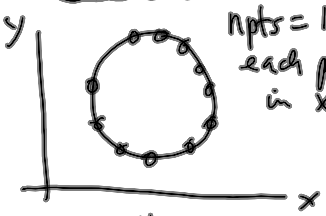
example where you need GLS

22-4

- fitting curves + surfaces where all coordinates are observed.
- coordinate transformation



npts = 11
each pt obs in x,y

$n = 11 \times 2 = 22$
$n_0 = 3 + 11 = 14$ *
$r = 8$

counting n_0: reconstruct figure w/ all obs.

$n_0 = 3 + 11$

to fix circle x_0, y_0, R

1 coordinate component per point

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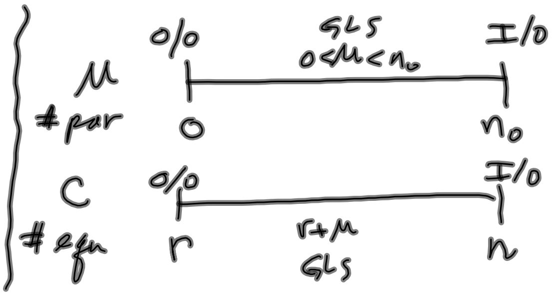
22-5

$n = 22$
$n_0 = 14$
$r = 8$

$\mu = 3$ $\quad X_c, Y_c, R$

$\boxed{C = r + \mu}$
$C = 8 + 3$
$C = 11$



$$(X_i - X_c)^2 + (Y_i - Y_c)^2 = R^2$$

$$\left[(X_i - X_c)^2 + (Y_i - Y_c)^2\right]^{1/2} = R$$

$$\boxed{F = \left[(X_i - X_c)^2 + (Y_i - Y_c)^2\right]^{1/2} - R = 0}$$

A, B

$$\frac{\partial F}{\partial X_i} = \frac{X_i - X_c}{[\cdot]^{1/2}}, \quad \frac{\partial F}{\partial Y_i} = \frac{Y_i - Y_c}{[\cdot]^{1/2}}$$

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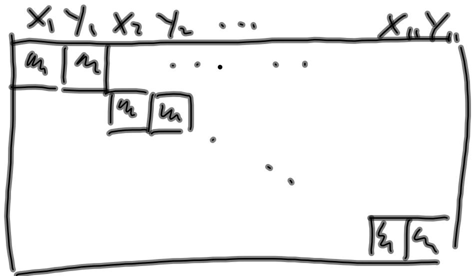
22-6

A



"Block Diagonal"

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