Homework 3 solution

1. 6 points, 4-par transformation, x, y – observed, XY – constants

$n = 12$ Indirect Observations (linear model)

$n_0 = 4$

$r = 8$ $\quad u = n_0 = 4 \quad a, b, c, d$

$$\begin{aligned} x &= aX + bY + c \\ y &= -bX + aY + d \end{aligned} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} X & Y & 1 & 0 \\ Y & -X & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \end{bmatrix} - \begin{bmatrix} X & Y & 1 & 0 \\ Y & -X & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} -X & -Y & -1 & 0 \\ -Y & X & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

$$V + B \cdot \Delta = f$$

$$\begin{bmatrix} v_{x_1} \\ v_{y_1} \\ v_{x_2} \\ v_{y_2} \\ \vdots \\ v_{x_6} \\ v_{y_6} \end{bmatrix} + \begin{bmatrix} -X_1 & -Y_1 & -1 & 0 \\ -Y_1 & X_1 & 0 & -1 \\ -X_2 & -Y_2 & -1 & 0 \\ -Y_2 & X_2 & 0 & -1 \\ & \vdots & & \\ -X_6 & -Y_6 & -1 & 0 \\ -Y_6 & X_6 & 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x_1 \\ -y_1 \\ -x_2 \\ -y_2 \\ \vdots \\ -x_6 \\ -y_6 \end{bmatrix}$$

$$V + B \cdot \Delta = f$$

$$\Delta = (B^T B)^{-1} B^T f$$

$$(W = I)$$

$$V = f - B\Delta$$

$$\Delta = \begin{bmatrix} 2.1154 \\ 0.5479 \\ 0.6960 \\ 0.8309 \end{bmatrix}$$

$$\lambda = \left[a^2 + b^2\right]^{1/2} = 2.1852$$

$$\cos\theta = a/\lambda, \quad \sin\theta = b/\lambda$$

$$\theta = \operatorname{atan}(\sin\theta / \cos\theta) = 0.2534 \text{ Rad} = 14.5149 \text{ deg}$$

$$V = \begin{bmatrix} -.0050 \\ -.0706 \\ .1648 \\ .0207 \\ -.0438 \\ .0924 \\ .0038 \\ .0391 \\ -.1444 \\ -.0350 \\ .0246 \\ -.0466 \end{bmatrix}$$

2. Same problem as (1) but now we write using physical parameters:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \lambda \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

this will be nonlinear in the 4 parameters: $\lambda, \theta, t_x, t_y$ so write as $F(\ell, x) = 0$,

$$F_x = x - \lambda\cos\theta X - \lambda\sin\theta Y - t_x = 0$$
$$F_y = y + \lambda\sin\theta X - \lambda\cos\theta Y - t_y = 0$$

now $\boxed{B = \partial F/\partial x}$ and $\boxed{f = -F(\ell, x^\circ)}$

use initial approximations: $\theta^\circ = 14^\circ \cdot (\pi/180)$

$\lambda^\circ = 2.0$

$t_x^\circ = 0.6$

$t_y^\circ = 0.8$

$$\begin{bmatrix} \partial F_x/\partial\lambda & \partial F_x/\partial\theta & \partial F_x/\partial t_x & \partial F_x/\partial t_y \\ \partial F_y/\partial\lambda & \partial F_y/\partial\theta & \partial F_y/\partial t_x & \partial F_y/\partial t_y \end{bmatrix} =$$

$$\begin{bmatrix} -\cos\theta X - \sin\theta Y & \sin\theta\lambda X - \cos\theta\lambda Y & -1 & 0 \\ \sin\theta X - \cos\theta Y & \cos\theta\lambda X + \sin\theta\lambda Y & 0 & -1 \end{bmatrix}$$

solution converges

$\lambda = 2.1852$

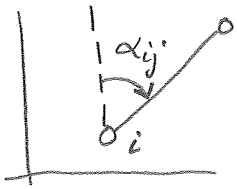
$\theta = 0.2534$

$t_x = 0.6960$

$t_y = 0.8309$

same as (1), residuals also same as (1)

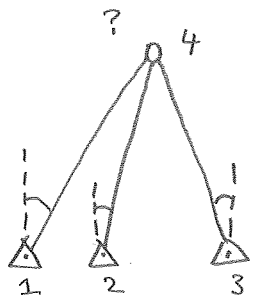
3.



$$\alpha = \text{atan}\left(\frac{x_j - x_i}{y_j - y_i}\right)$$

$$F_\alpha = \alpha - \text{atan}\left(\Delta x / \Delta y\right) = 0$$

$$\frac{\partial F_\alpha}{\partial x_j} = \frac{-\Delta y}{D^2}, \quad \frac{\partial F_\alpha}{\partial y_j} = \frac{\Delta x}{D^2}$$



$n = 3$ Indirect Observations, $u = n_0 = 2$: X, Y of unknown point

$n_0 = 2$

$r = 1$

$$B = \begin{bmatrix} -\frac{\Delta y_{14}}{D_{14}^2} & \frac{\Delta x_{14}}{D_{14}^2} \\ -\frac{\Delta y_{24}}{D_{24}^2} & \frac{\Delta x_{24}}{D_{24}^2} \\ -\frac{\Delta y_{34}}{D_{34}^2} & \frac{\Delta x_{34}}{D_{34}^2} \end{bmatrix}, \quad f = -F_\alpha(l, x^\circ)$$

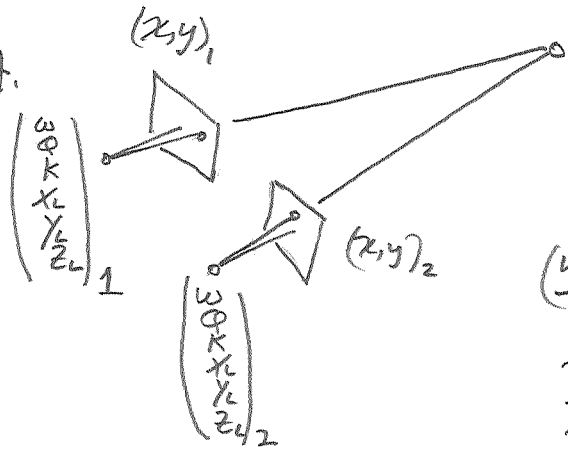
$(X^\circ, Y^\circ) = (1.8, 3.9)$ from sketch

solution converges ✓

$$V = \begin{bmatrix} -.0041 \\ .0052 \\ -.0017 \end{bmatrix} \text{Rad}, \quad V = \begin{bmatrix} -.2333 \\ .2968 \\ -.0989 \end{bmatrix} \text{deg.}$$

$$\begin{pmatrix} \hat{X} \\ \hat{Y} \end{pmatrix} = \begin{pmatrix} 1.9999 \\ 3.9701 \end{pmatrix}$$

4.



all variables shown in sketch are observations, $n = 19$

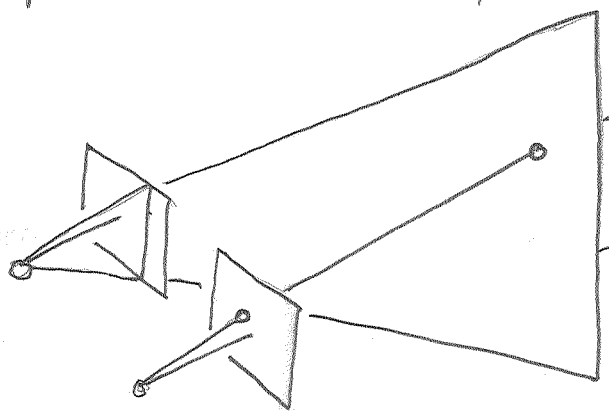
if we have $(\underline{\omega}, \underline{\varphi}, \underline{\kappa}, \underline{X_L}, \underline{Y_L}, \underline{Z_L})_1$ and $(\underline{\omega}, \underline{\varphi}, \underline{\kappa}, \underline{X_L}, \underline{Y_L}, \underline{Z_L})_2$ & (x_0, y_0, f) fixed, and

$\underline{x_1}$ then defines a plane in space

$\underline{x_2}, \underline{y_2}$ then defines a ray in space which intersects the plane uniquely. $n_0 = 15$.

$n - n_0 = r = 4$.

Therefore the observation only model will require 4 condition equations.



two coordinates on one photo define a ray in space

one coordinate on one photograph defines a plane

eliminate λ, and 1 equation

$$\begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \\ -foc \end{bmatrix} = \lambda_1 M_1 \begin{bmatrix} x - x_{L_1} \\ Y - Y_{L_1} \\ Z - Z_{L_1} \end{bmatrix} = \lambda_1 \begin{bmatrix} u \\ v \\ w \end{bmatrix}_1 \quad , \quad \begin{aligned} F_{x_1} &= x_1 + foc\,(u_1/w_1) = 0 \\ F_{y_1} &= y_1 + foc\,(v_1/w_1) = 0 \end{aligned}$$

$$\begin{bmatrix} x_2 - x_0 \\ y_2 - y_0 \\ -foc \end{bmatrix} = \lambda_2 M_2 \begin{bmatrix} x - x_{L_2} \\ Y - Y_{L_2} \\ Z - Z_{L_2} \end{bmatrix} = \lambda_2 \begin{bmatrix} u \\ v \\ w \end{bmatrix}_2 \quad , \quad \begin{aligned} F_{x_2} &= x_2 + foc\,(u_2/w_2) = 0 \\ F_{y_2} &= y_2 + foc\,(v_2/w_2) = 0 \end{aligned}$$

for observations only $A v = f$

$$A = \begin{bmatrix} \frac{\partial F_{x_1}}{\partial \omega_1} & \frac{\partial F_{x_1}}{\partial \varphi_1} & \frac{\partial F_{x_1}}{\partial \kappa_1} & \frac{\partial F_{x_1}}{\partial X_{L_1}} & \frac{\partial F_{x_1}}{\partial Y_{L_1}} & \frac{\partial F_{x_1}}{\partial Z_{L_1}} & \frac{\partial F_{x_1}}{\partial \omega_2} & \cdots & \frac{\partial F_{x_1}}{\partial X} & \frac{\partial F_{x_1}}{\partial Y} & \frac{\partial F_{x_1}}{\partial Z} & \cdots & \frac{\partial F_{x_1}}{\partial x_2} & \frac{\partial F_{x_1}}{\partial y_2} \\ \frac{\partial F_{y_1}}{\partial \omega_1} & & & & & & & & & & & & & \vdots \\ \frac{\partial F_{x_2}}{\partial \omega_1} & & & & & & & & & & & & & \\ \frac{\partial F_{y_2}}{\partial \omega_1} & \cdots & & & & & & \cdots & & & & & \frac{\partial F_{y_2}}{\partial x_2} & \frac{\partial F_{y_2}}{\partial y_2} \end{bmatrix}$$

(4,19)

(evaluate numerically)

$$f = \begin{bmatrix} -F_{x_1} \\ -F_{y_1} \\ -F_{x_2} \\ -F_{y_2} \\ \vdots \end{bmatrix} - A(l - l^\circ)$$

$\uparrow$ evaluated at l°

A: eval. at l°

l°: original observations

l: current updated observations

$$V = \begin{bmatrix} .0003 \\ .0001 \\ .0001 \\ -.0029 \\ .0001 \\ .0092 \\ -.0028 \\ .0108 \\ -.0003 \\ -.0001 \\ .0001 \\ .0023 \\ .0009 \\ -.0080 \\ .0025 \\ -.0102 \\ 1.0455 \\ -1.6329 \\ -1.9331 \end{bmatrix} \qquad \hat{l} = \begin{bmatrix} 1.5545 \\ .0350 \\ .0514 \\ 10.0071 \\ 20.0201 \\ 44.9792 \\ 3.3422 \\ .7788 \\ 1.5710 \\ -.0172 \\ .0350 \\ 60.0323 \\ 9.9509 \\ 45.0320 \\ -4.0175 \\ .7728 \\ 30.0455 \\ 129.3671 \\ 50.0669 \end{bmatrix}$$

ω_1
ϕ_1
κ_1
X_{L_1}
Y_{L_1}
Z_{L_1}
x_1
y_1
ω_2
ϕ_2
κ_2
X_{L_2}
Y_{L_2}
Z_{L_2}
x_2
y_2
X
Y
Z

Order ↓

$$W = \begin{bmatrix} 1/\sigma^2_{\omega_1} & & \\ & 1/\sigma^2_{\phi_1} & \\ & & \ddots \end{bmatrix}$$

note order of weights must be same as order of columns in A

```
hw123

hw3 problem #1
dl =
    2.1154
    0.5479
    0.6960
    0.8309
v =
   -0.0050
   -0.0706
    0.1648
    0.0207
   -0.0438
    0.0924
    0.0038
    0.0391
   -0.1444
   -0.0350
    0.0246
   -0.0466
lam =
    2.1852
cos_th =
    0.9681
sin_th =
    0.2507
theta =
    0.2534
thetad =
   14.5194
tx =
    0.6960
ty =
    0.8309

hw3 problem #2
ans =
    0.1851    0.0099    0.0960   -0.0691
ans =
   1.0e-03 *
    0.0890   -0.8394   -0.0000    0.0000
ans =
   1.0e-06 *
    0.7698    0.0344    0.0000   -0.0000
ans =
   1.0e-13 *
    0.0129   -0.1206   -0.0004    0.0050
we have converged
v =
   -0.0050
   -0.0706
    0.1648
    0.0207
   -0.0438
    0.0924
    0.0038
    0.0391
   -0.1444
   -0.0350
    0.0246
   -0.0466
lam =
    2.1852
theta =
    0.2534
thetad =
   14.5194
tx =
    0.6960
ty =
    0.8309

hw3 problem #3
ans =
    0.1968    0.0762
ans =
    0.0031   -0.0062
ans =
```

```
   1.0e-04 *
   -0.0280    0.2730
ans =
   1.0e-07 *
    0.0296   -0.5243
ans =
   1.0e-10 *
   -0.0756    0.9067
we have converged
v =
   -0.0041
    0.0052
   -0.0017
vd =
   -0.2333
    0.2968
   -0.0989
ans =
    1.9999    3.9701
diary off
```

```
hw3_4
problem 4 solution (2 photo intersection)
wt =
   1.0e+06 *
    1.3131
    1.3131
    1.3131
    0.0004
    0.0004
    0.0004
    0.0025
    0.0025
    1.3131
    1.3131
    1.3131
    0.0004
    0.0004
    0.0004
    0.0025
    0.0025
    0.0000
    0.0000
    0.0000
W =
   1.0e+06 *
  Columns 1 through 12
    1.3131         0         0         0         0         0         0         0         0         0
   0         0
         0    1.3131         0         0         0         0         0         0         0         0
   0         0
         0         0    1.3131         0         0         0         0         0         0         0
   0         0
         0         0         0    0.0004         0         0         0         0         0         0
   0         0
         0         0         0         0    0.0004         0         0         0         0         0
   0         0
         0         0         0         0         0    0.0004         0         0         0         0
   0         0
         0         0         0         0         0         0    0.0025         0         0         0
   0         0
         0         0         0         0         0         0         0    0.0025         0         0
   0         0
         0         0         0         0         0         0         0         0    1.3131         0
   0         0
         0         0         0         0         0         0         0         0         0    1.3131
   0         0
         0         0         0         0         0         0         0         0         0         0
1.3131         0
         0         0         0         0         0         0         0         0         0         0
   0    0.0004
         0         0         0         0         0         0         0         0         0         0
   0         0
         0         0         0         0         0         0         0         0         0         0
   0         0
         0         0         0         0         0         0         0         0         0         0
   0         0
         0         0         0         0         0         0         0         0         0         0
   0         0
         0         0         0         0         0         0         0         0         0         0
   0         0
         0         0         0         0         0         0         0         0         0         0
   0         0
         0         0         0         0         0         0         0         0         0         0
   0         0
  Columns 13 through 19
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
         0         0         0         0         0         0         0
    0.0004         0         0         0         0         0         0
         0    0.0004         0         0         0         0         0
```

```
         0         0    0.0025         0         0         0         0
         0         0         0    0.0025         0         0         0
         0         0         0         0    0.0000         0         0
         0         0         0         0         0    0.0000         0
         0         0         0         0         0         0    0.0000

check convergence by visual inspection

ans =
  Columns 1 through 12
    0.0003    0.0001    0.0001   -0.0031   -0.0001    0.0092   -0.0031    0.0110   -0.0003   -0.0001
0.0001    0.0024
  Columns 13 through 19
    0.0011   -0.0080    0.0027   -0.0103    1.0606   -1.6465   -1.9602
ans =
  Columns 1 through 12
   -0.0000   -0.0000    0.0000    0.0002    0.0002    0.0000    0.0002   -0.0002    0.0000    0.0000
-0.0000   -0.0002
  Columns 13 through 19
   -0.0002   -0.0000   -0.0002    0.0001   -0.0151    0.0134    0.0271
ans =
   1.0e-03 *
  Columns 1 through 12
    0.0000    0.0000   -0.0000   -0.0014   -0.0024   -0.0010   -0.0022    0.0002   -0.0000   -0.0001
0.0000    0.0014
  Columns 13 through 19
    0.0023    0.0010    0.0021    0.0000   -0.0018    0.1202    0.0088
ans =
   1.0e-06 *
  Columns 1 through 12
    0.0001    0.0002   -0.0000   -0.0024   -0.0001   -0.0083   -0.0067    0.0026    0.0000   -0.0002
-0.0001    0.0024
  Columns 13 through 19
   -0.0001    0.0083    0.0057    0.0005   -0.0128    0.3249    0.0300
ans =
   1.0e-08 *
  Columns 1 through 12
    0.0000    0.0001   -0.0000   -0.0009   -0.0001   -0.0022   -0.0020    0.0006    0.0000   -0.0001
-0.0000    0.0009
  Columns 13 through 19
    0.0001    0.0022    0.0019    0.0001   -0.0029    0.1046    0.0081
v =
    0.0003
    0.0001
    0.0001
   -0.0029
    0.0001
    0.0092
   -0.0028
    0.0108
   -0.0003
   -0.0001
    0.0001
    0.0023
    0.0009
   -0.0080
    0.0025
   -0.0102
    1.0455
   -1.6329
   -1.9331
v_deg =
    0.0176
    0.0056
    0.0042
   -0.0029
    0.0001
    0.0092
   -0.0028
    0.0108
   -0.0167
   -0.0052
    0.0043
    0.0023
    0.0009
   -0.0080
    0.0025
   -0.0102
    1.0455
```

```
   -1.6329
   -1.9331
lhat =
    1.5545
    0.0350
    0.0514
   10.0071
   20.0201
   44.9792
    3.3422
    0.7788
    1.5710
   -0.0172
    0.0350
   60.0323
    9.9509
   45.0320
   -4.0175
    0.7728
   30.0455
  129.3671
   50.0669
lhat_deg =
   89.0676
    2.0056
    2.9442
   10.0071
   20.0201
   44.9792
    3.3422
    0.7788
   90.0133
   -0.9852
    2.0043
   60.0323
    9.9509
   45.0320
   -4.0175
    0.7728
   30.0455
  129.3671
   50.0669
diary off

order of observations (19 of them):
w1,p1,k1,XL1,YL1,ZL1,x1,y1,w2,p2,k2,XL2,YL2,ZL2,x2,y2,X,Y,Z
```

```
% hw3_123.m  14-oct-2013
% slove hw3 problems 1,2,3

disp('hw3 problem #1');
n=12;
n0=4;
r=8;

B=zeros(12,4);
f=zeros(12,1);
W=eye(12);

X=[1;2;2;3;4;5];
Y=[3;1;5;4;1;4];
x=[4.46;5.31;7.71;9.23;9.85;13.44];
y=[6.70;1.83;10.22;7.61;0.79;6.60];

for i=1:6
  XX=X(i);
  YY=Y(i);
  xx=x(i);
  yy=y(i);
  ii=(i-1)*2 + 1;
  B(ii:ii+1,:)=[-XX -YY -1 0;-YY XX 0 -1];
  f(ii:ii+1)=[-xx;-yy];
  end

dl=inv(B'*B)*B'*f
v=f-B*dl

a=dl(1);
b=dl(2);
c=dl(3);
d=dl(4);

lam=sqrt(a^2 + b^2)
cos_th=a/lam
sin_th=b/lam
theta=atan(sin_th/cos_th)
degrad=180/pi;
thetad=theta*degrad
tx=c
ty=d

%====================================%
%====================================%
%===== now do nonlinear version =====%
%====================================%
%====================================%

disp(' ');
disp('hw3 problem #2');

theta=14/degrad;
lam=2.0;
tx=0.6;
ty=0.9;

n_iter=0;
keep_going=1;
while(keep_going==1)
  % LS code
  B=zeros(12,4);
  f=zeros(12,1);

  for i=1:6
    XX=X(i);
    YY=Y(i);
    xx=x(i);
    yy=y(i);
    ii=(i-1)*2 + 1;
    B(ii,1)=-cos(theta)*XX - sin(theta)*YY;
    B(ii,2)=sin(theta)*lam*XX - cos(theta)*lam*YY;
    B(ii,3)=-1;
    B(ii,4)=0;
    B(ii+1,1)=sin(theta)*XX - cos(theta)*YY;
    B(ii+1,2)=cos(theta)*lam*XX + sin(theta)*lam*YY;
    B(ii+1,3)=0;
    B(ii+1,4)=-1;
```

```
    f(ii)=  -(xx - lam*cos(theta)*XX - lam*sin(theta)*YY - tx);
    f(ii+1)=-(yy + lam*sin(theta)*XX - lam*cos(theta)*YY - ty);
    end

  dl=inv(B'*B)*B'*f;
  dl'
  lam=lam + dl(1);
  theta=theta + dl(2);
  tx=tx + dl(3);
  ty=ty + dl(4);

  if all(abs(dl) < 1e-08)
    keep_going=0;
    disp('we have converged');
    v=f-B*dl
    lam
    theta
    thetad=theta*degrad
    tx
    ty
    end
  if(n_iter > 10)
    keep_going=0;
    disp('we did not converge');
    end
  n_iter=n_iter + 1;
  end


%====================================%
%====================================%
%========= now do problem 3 =========%
%====================================%
%====================================%

disp(' ');
disp('hw3 problem #3');

% n=3
% n0=2
% r=1
% unknowns x,y

alph=[26.97/degrad; 13.84/degrad; -14.04/degrad];
X=[0;1;3;1.8];
Y=[0;0;0;3.9];

n_iter=0;
keep_going=1;
while(keep_going==1)
  % LS code
  B=zeros(3,2);
  f=zeros(3,1);
  for i=1:3
    dx=X(4)-X(i);
    dy=Y(4)-Y(i);
    Dsqr=dx^2 + dy^2;
    dFdXj=-dy/Dsqr;
    dFdYj=dx/Dsqr;
    B(i,:)=[dFdXj dFdYj];
    F=alph(i) - atan(dx/dy);
    f(i)=-F;
    end

  dl=inv(B'*B)*B'*f;
  dl'
  X(4)=X(4)+dl(1);
  Y(4)=Y(4)+dl(2);

  if all(abs(dl) < 1e-08)
    keep_going=0;
    disp('we have converged');
    v=f-B*dl
    vd=v*degrad
    [X(4) Y(4)]
    end
  if(n_iter > 10)
    keep_going=0;
    disp('we did not converge');
```

```
  end
n_iter=n_iter + 1;
end
```

```
% hw3_4.m  5-oct-2013
% solve photogrammetry obs. only problem
% n=19, n0=15, r=4
% need c=4 condition equations (collinearity for one
% point on each photo)

degrad=180/pi;

w1=89.05/degrad;
p1=2.00/degrad;
k1=2.94/degrad;
XL1=10.01;
YL1=20.02;
ZL1=44.97;
x1=3.345;
y1=0.768;

w2=90.03/degrad;
p2=-0.98/degrad;
k2=2.00/degrad;
XL2=60.03;
YL2=9.95;
ZL2=45.04;
x2=-4.020;
y2=0.783;

X=29.0;
Y=131.0;
Z=52.0;

x0=0;
y0=0;
foc=15.00;

% project rays into XZ plane at Y-location of the point
% try to explain the odd residuals (all corrections to photo 2)

M=m3(k1)*m2(p1)*m1(w1);
uvw=M'*[x1;y1;-foc];
XX=XL1 + (Y-YL1)*uvw(1)/uvw(2);
ZZ=ZL1 + (Y-YL1)*uvw(3)/uvw(2);
M=m3(k2)*m2(p2)*m1(w2);
uvw=M'*[x2;y2;-foc];
XX=XL2 + (Y-YL2)*uvw(1)/uvw(2);
ZZ=ZL2 + (Y-YL2)*uvw(3)/uvw(2);

l=[w1;p1;k1;XL1;YL1;ZL1;x1;y1;w2;p2;k2;XL2;YL2;ZL2;x2;y2;X;Y;Z];
l0=l;
A=zeros(4,19);
F=zeros(4,1);
f=zeros(4,1);
s=[0.05/degrad;0.05/degrad;0.05/degrad;0.05;0.05;0.05;0.02;0.02;
   0.05/degrad;0.05/degrad;0.05/degrad;0.05;0.05;0.05;0.02;0.02;
   2.0;2.0;2.0];
ssqr=s.^2;
wt=(1.0)./ssqr;
W=diag(wt);
wt
W

% lazy convergence testing
for i=1:5
  [F1,J1]=lincol(w1,p1,k1,XL1,YL1,ZL1,X,Y,Z,x1,y1,x0,y0,foc);
  A(1:2,1:6)=J1(1:2,1:6);
  A(1:2,7:8)=J1(1:2,10:11);
  A(1:2,17:19)=J1(1:2,7:9);
  F(1:2)=F1;
  [F2,J2]=lincol(w2,p2,k2,XL2,YL2,ZL2,X,Y,Z,x2,y2,x0,y0,foc);
  A(3:4,9:14)=J2(1:2,1:6);
  A(3:4,15:16)=J2(1:2,10:11);
  A(3:4,17:19)=J2(1:2,7:9);
  F(3:4)=F2;
  f=-F - A*(l-l0);
  Q=inv(W);
  Qe=A*Q*A';
  We=inv(Qe);
  kk=We*f;
  v=Q*A'*kk;
  l0_new=l+v;
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  delta_l=l0_new - l0;
  l0=l0_new;
  w1=l0(1);
  p1=l0(2);
  k1=l0(3);
  XL1=l0(4);
  YL1=l0(5);
  ZL1=l0(6);
  x1=l0(7);
  y1=l0(8);
  w2=l0(9);
  p2=l0(10);
  k2=l0(11);
  XL2=l0(12);
  YL2=l0(13);
  ZL2=l0(14);
  x2=l0(15);
  y2=l0(16);
  X=l0(17);
  Y=l0(18);
  Z=l0(19);
  delta_l'
  end

pause
v

v_deg=v;
v_deg(1:3)=v_deg(1:3)*degrad;
v_deg(9:11)=v_deg(9:11)*degrad;
v_deg

lhat=l+v
lhat_deg=lhat;
lhat_deg(1:3)=lhat_deg(1:3)*degrad;
lhat_deg(9:11)=lhat_deg(9:11)*degrad;
lhat_deg
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% lincol.m  4-oct-2013
% linearize collinearity equations
% get partial derivatives of w,p,k,XL,YL,ZL,X,Y,Z,x,y

function [F,J]=lincol(w,p,k,XL,YL,ZL,X,Y,Z,x,y,x0,y0,foc)

P=[w;p;k;XL;YL;ZL;X;Y;Z;x;y;x0;y0;foc];
dP=[1e-05;1e-05;1e-05;1e-04;1e-04;1e-04;1e-04;1e-04;1e-04;1e-03;1e-03;1e-03;1e-03;1e-03];

dFdP=zeros(2,14);
F0=col(P);
for i=1:14
  WP=P;
  WP(i)=WP(i)+dP(i);
  F1=col(WP);
  dFdP(:,i)=(F1-F0)/dP(i);
  end

% returned values

F=F0;
J=dFdP;
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% col.m  4-oct-2013
% evaluate collinearity equations

function F=col(P)

w=P(1);
p=P(2);
k=P(3);
XL=P(4);
YL=P(5);
ZL=P(6);
X=P(7);
Y=P(8);
Z=P(9);
x=P(10);
y=P(11);
x0=P(12);
y0=P(13);
foc=P(14);

M=m3(k)*m2(p)*m1(w);
UVW=M*[X-XL; Y-YL;Z-ZL];
F=[x - x0 + foc*(UVW(1)/UVW(3));
   y - y0 + foc*(UVW(2)/UVW(3))];
```