CE 597 Adjustment of Geospatial Observations
Exam 1

NAME ____________________

Calculator + 1 page of notes allowed (both sides), no phones, tablets, no internet

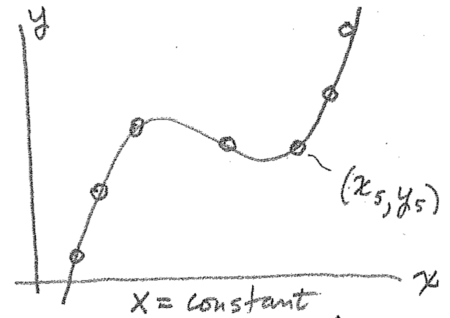
SHOW ALL YOUR WORK

1. 7 points are observed on a curve with equation $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$. You wish to estimate, by LS, the parameters, $a_i$, of the polynomial by indirect observations.

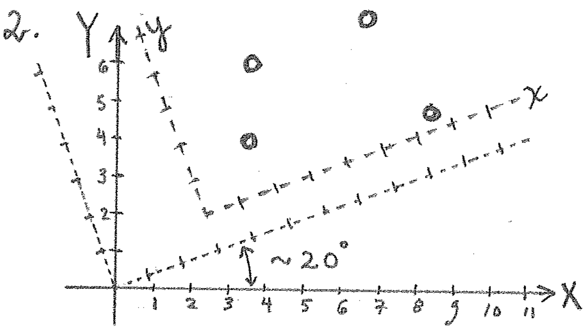
(a) what are $n$, $n_0$, and $r$ ?

(b) would you handle as <u>linear</u> or <u>nonlinear</u> ?

(c) show 1 condition equation in form, $v + B\Delta = f$




X = constant
Y = observation

2. 


extra axes are parallel to x,y.

4 points are observed in x,y. X,Y are constant. You must solve an indirect observation, LS problem, for $\theta, t_x, t_y$, using equations,

$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

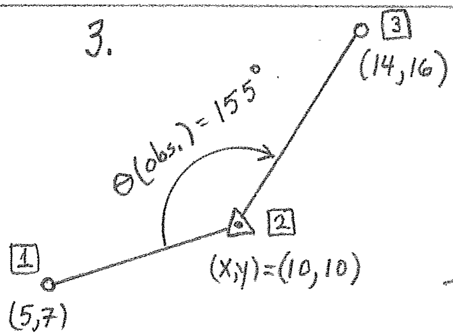
$\lambda$ is <u>fixed</u> at 1.0000, a constant.

(a) what are $n$, $n_0$, and $r$ ?

(b) what is the size of the normal equation coefficient matrix ?

(c) what are approximate values for $\theta, t_x, t_y$ (by visual inspection of figure) ?

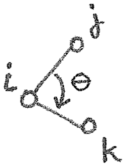
(d) is there a substitution you could make, so the problem becomes <u>linear</u> ? if yes, what is it ?

3. 


You observe the angle at control point 2 as 155°. Points 1 and 3 are unknowns to be solved for. The condition equation is : $F_\theta = \theta_{ijk} - \text{computed } \theta = 0$

partial derivatives are given below. Show <u>numerical</u> values for $\underline{B} = \left[ \frac{\partial F}{\partial X_1} \ \frac{\partial F}{\partial Y_1} \ \frac{\partial F}{\partial X_3} \ \frac{\partial F}{\partial Y_3} \right]$, and $\underline{f}$ of the linearized condition equation. Approximate coordinates are given.

useful info.




$\Delta X_{ij} = X_j - X_i$

$\Delta Y_{ij} = Y_j - Y_i$

$\Delta X_{ik} = X_k - X_i$

$\Delta Y_{ik} = Y_k - Y_i$

$\dfrac{\partial F_\theta}{\partial X_i} = \dfrac{\Delta Y_{ik}}{D_{ik}^2} - \dfrac{\Delta Y_{ij}}{D_{ij}^2}$

$\dfrac{\partial F_\theta}{\partial X_j} = \dfrac{\Delta Y_{ij}}{D_{ij}^2}$

$\dfrac{\partial F_\theta}{\partial X_k} = -\dfrac{\Delta Y_{ik}}{D_{ik}^2}$

$\dfrac{\partial F_\theta}{\partial Y_i} = -\dfrac{\Delta X_{ik}}{D_{ik}^2} + \dfrac{\Delta X_{ij}}{D_{ij}^2}$

$\dfrac{\partial F_\theta}{\partial Y_j} = -\dfrac{\Delta X_{ij}}{D_{ij}^2}$

$\dfrac{\partial F_\theta}{\partial Y_k} = \dfrac{\Delta X_{ik}}{D_{ik}^2}$

| Quadr. | ΔX | ΔY | atan(ΔX/ΔY) | transf. | atan2(ΔX,ΔY) |
|---|---|---|---|---|---|
| I | + | + | $\gamma$ | → | $\alpha$ |
| II | − | + | $\gamma$ | → | $\alpha$ |
| III | − | − | $\gamma$ | $\gamma - 180$ | $\alpha$ |
| IV | + | − | $\gamma$ | $\gamma + 180$ | $\alpha$ |
| | | | | | azimuth |

# Exam 1 Solution (Mean score = 83)

(1.)


$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

(a) $n = 7$
$\underline{n_0 = 4}$
$r = 3$

(b) linear
(x is constant)

(c) $y + v_y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$, $\quad v_y - a_0 - a_1 x - a_2 x^2 - a_3 x^3 = -y$

$$[v_y] + \begin{bmatrix} -1 & -x & -x^2 & -x^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = [-y] \quad , \quad v + B\Delta = f$$

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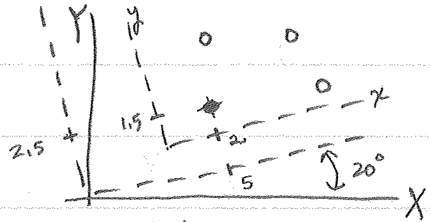
(2.)




$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

$\lambda = 1.0000$ fixed constant

Solve for $\theta, t_x, t_y$

$x, y$ : observations
$X, Y$ : constants

(a) $n = 8$
$\underline{n_0 = 3}$
$r = 5$

(b) normal equation matrix, $N$: 3x3

(c) $\theta \approx 20°$
$t_x \approx -3$
$t_y \approx -1$

[Did not grade this one]
(d) no substitution possible unless you change the problem!

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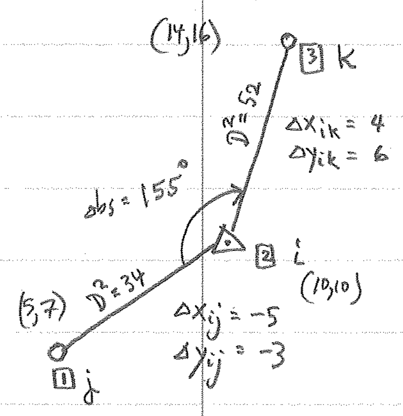
(3.) $F_\theta = \underset{(obs)}{\theta_{ijk}} - \underset{(computed)}{\theta_{ijk}} = 0$ ,




$$B = \begin{bmatrix} \dfrac{\partial F}{\partial x_j} & \dfrac{\partial F}{\partial y_j} & \dfrac{\partial F}{\partial x_k} & \dfrac{\partial F}{\partial y_k} \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{\Delta y_{ij}}{D_{ij}^2} & \dfrac{-\Delta x_{ij}}{D_{ij}^2} & \dfrac{-\Delta y_{ik}}{D_{ik}^2} & \dfrac{\Delta x_{ik}}{D_{ik}^2} \end{bmatrix}$$

$$B = \begin{bmatrix} \dfrac{-3}{34} & \dfrac{5}{34} & \dfrac{-6}{52} & \dfrac{4}{52} \end{bmatrix}$$

$$B = \begin{bmatrix} -.088 & .147 & -.115 & .077 \end{bmatrix}$$

$$F_\theta = \theta_{ijk} - [az_{ik} - az_{ij}] = 0$$

$$= \theta_{ijk} - \left[ \operatorname{atan}\left(\frac{\Delta x_{ik}}{\Delta y_{ik}}\right) - \operatorname{atan}\left(\frac{\Delta x_{ij}}{\Delta y_{ij}}\right) \right]$$

$$= \theta_{ijk} - \underbrace{\left[ \operatorname{atan}\left(\frac{4}{6}\right) - \left( \operatorname{atan}\left(\frac{-5}{-3}\right) - 180° \right) \right]}_{\substack{33.69 \ - \ (59.036 \ - \ 180) \\ 154.65°}}$$

$$F_\theta = 155° - 154.65° = 0.35°$$

$$f = -F = -0.35°$$

$$= -0.00604 \text{ Radians}$$