$\Phi = v_1^2 + v_2^2 + v_3^2$ $\quad y_i + v_i = mx_i + b$

4-1

$\Phi = (m+b-1)^2 + (m\cdot 2+b-2.2)^2 + (m\cdot 3+b-3)^2$

$\frac{\partial \Phi}{\partial m} = 2(m+b-1) + 2(m\cdot 2+b-2.2)(2) + 2(m\cdot 3+b-3)(3) = 0$

$\frac{\partial \Phi}{\partial b} = 2(m+b-1) + 2(m\cdot 2+b-2.2) + 2(m\cdot 3+b-3) = 0$

$m + 4m + 9m + b + 2b + 3b = 1 + 4.4 + 9$

$m + 2m + 3m + b + b + b = 1 + 2.2 + 3$

$$\begin{aligned} 14m + 6b &= 14.4 \\ 6m + 3b &= 6.2 \end{aligned}$$

normal equations

$$\begin{bmatrix} 14 & 6 \\ 6 & 3 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 14.4 \\ 6.2 \end{bmatrix}$$

N p t

coefficient matrix

SYMMETRIC

full rank

```
N = [14 6; 6 3];
t = [14.4; 6.2];
p = inv(N) * t
```

$p = \begin{bmatrix} 1.0000 \\ 0.0667 \end{bmatrix}$ m = slope, b = intercept

for max 2x2 systems – solve by hand cramers rule, matrix inverse, gauss elim.

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LS estimates $\hat{m}, \hat{b}$ $\quad y_i + v_i = mx_i + b$

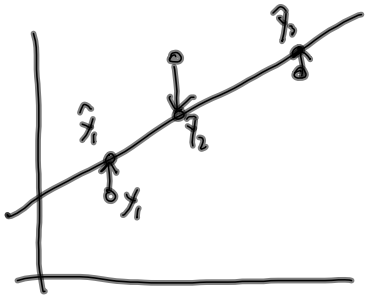
4-2

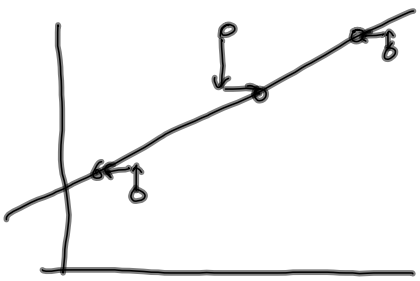
$v_i = mx_i + b - y_i$

$v_1 = mx_1 + b - y_1 = 0.0667$

$v_2 = mx_2 + b - y_2 = -0.1333$ *

$v_3 = mx_3 + b - y_3 = 0.0667$





$\hat{y}_1 = y_1 + v_1$ $\qquad \hat{y}_i = mx_i + b$

* note: this value was wrong in original notes. I corrected here, I will not try to correct the video file.

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$v_1 = +1^{\circ}$
$v_2 = +1^{\circ}$
$v_3 = +1^{\circ}$

hard solution of linear fit problem, indirect observations

4-3

- choose n_0 parameters
- write n cond. eqns, each obs expressed as function of those parameters

Solve same problem by observations only

$n = 3$
$n_0 = 2$
$r = 1$

no parameters
write exactly r condition eqn's.

$\hat{y}_1, \hat{y}_2, \hat{y}_3 \quad x_1, x_2, x_3$

1 cond. eqn ?

$$\frac{\hat{y}_2 - \hat{y}_1}{x_2 - x_1} = \frac{\hat{y}_3 - \hat{y}_1}{x_3 - x_1}$$

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