Derive LS solution Matrix methods I/O

obj function $\sum v_i^2 \to \sum w_i v_i^2$ 6-1

$\hat{y}_i = m x_i + b$, $y_i + v_i = m x_i + b$

$(v + B\Delta = f)$ $v_i - m x_i - b = -y_i$

$\phi = v^T W v$

$$\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} w_1 & 0 & 0 \\ 0 & w_2 & 0 \\ 0 & 0 & w_3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$$

$1n$ $n1$

$v_1 - m x_1 - b = -y_1$
$v_2 - m x_2 - b = -y_2$
$v_3 - m x_3 - b = -y_3$

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -x_1 & -1 \\ -x_2 & -1 \\ -x_3 & -1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -y_1 \\ -y_2 \\ -y_3 \end{bmatrix}$$

$$\underset{(n,1)}{v} + \underset{(c,u)}{B}\ \underset{(u,1)}{\Delta} = \underset{(n,1)}{f}$$

$c = n$, $u = n_0$

other notations

$Ax = b + e$

$z = Hx + f$

$v = f - B\Delta$

$$\phi = (f - B\Delta)^T W (f - B\Delta)$$
n,n $n,1$

$$\phi = (f^T - \Delta^T B^T) W (f - B\Delta)$$

$$\phi = f^T W f - f^T W B\Delta - \underbrace{\Delta^T B^T W f}_{= f^T W B\Delta} + \Delta^T B^T W B \Delta$$

$$\phi = f^T W f - 2 f^T W B\Delta + \Delta^T B^T W B \Delta$$

need derivative w.r.t. vector

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$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \vec{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \end{bmatrix}$$

$\vec{y} = A\vec{x}$ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$

$$a: \quad \frac{da}{\partial \vec{x}} = \begin{bmatrix} \frac{da}{\partial x_1} & \frac{da}{\partial x_2} & \frac{da}{\partial x_3} \end{bmatrix} \checkmark$$

$$\frac{d\vec{y}}{dx} = \begin{bmatrix} dy_1/dx_1 & dy_1/dx_2 & dy_1/dx_3 \\ dy_2/dx_1 & dy_2/dx_2 & dy_2/dx_3 \end{bmatrix}$$

$$\vec{y} = A\vec{x}, \quad \frac{d\vec{y}}{d\vec{x}} = A \quad \text{I}$$

need $\frac{d}{dx} x^T A x$ A: sym.

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} & x_1 a_{12} + x_2 a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$x_1^2 a_{11} + x_1 x_2 a_{12} + x_1 x_2 a_{12} + x_2^2 a_{22}$$

$$x_1^2 a_{11} + 2 x_1 x_2 a_{12} + x_2^2 a_{22}$$

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$$\frac{d}{d\vec{x}}\left(x_1^2 a_{11} + 2x_1x_2 a_{12} + x_2^2 a_{22}\right) =$$

$$\begin{bmatrix} 2x_1 a_{11} + 2x_2 a_{12} & 2x_1 a_{12} + 2x_2 a_{22} \end{bmatrix}$$

$$2\begin{bmatrix} x_1 a_{11} + x_2 a_{12} & x_1 a_{12} + x_2 a_{22} \end{bmatrix}$$

$$2\left[\begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix} \quad \begin{bmatrix} x_1 & x_2 \end{bmatrix}\begin{bmatrix} a_{12} \\ a_{22} \end{bmatrix} \right]$$

$$2\begin{bmatrix} x_1 & x_2 \end{bmatrix}\left[\begin{pmatrix} a_{11} \\ a_{12} \end{pmatrix} \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}\right] = 2x^TA$$

$$\frac{d}{d\vec{x}} x^TAx = 2x^TA \quad \text{II}$$

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$$\phi = f^TWf - \underset{\text{I}}{2f^TWB\Delta} + \Delta^T \underbrace{B^TWB}_{\text{II}} \Delta$$

$$\frac{d\phi}{\partial \vec{\Delta}} = -\cancel{2}f^TWB + \cancel{2}\Delta^TB^TWB = 0$$

$$-B^TWf + B^TWB\Delta = 0$$

$$\underbrace{B^TWB}_{N}\Delta = \underbrace{B^TWf}_{t} \quad \text{Normal Eqns}$$

$$N\Delta = t$$

$$\Delta = N^{-1}t \quad \text{solution parameter vector}$$

$V = f - B\Delta$ compute residuals

$\hat{y} = y + v$

Solve problem: analyze n, n_0, r (w)
write n cond. eqns $V + B\Delta = f$
$\Delta = (B^TWB)^{-1}B^TWf$

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