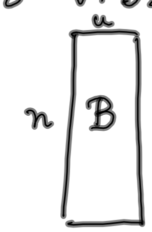
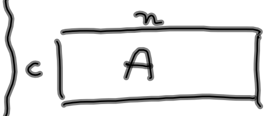
I/O $v + B s = f$ | O/O $Av = f$



rank(B) = u

$u = n_0$



rank(A) = c

c = r

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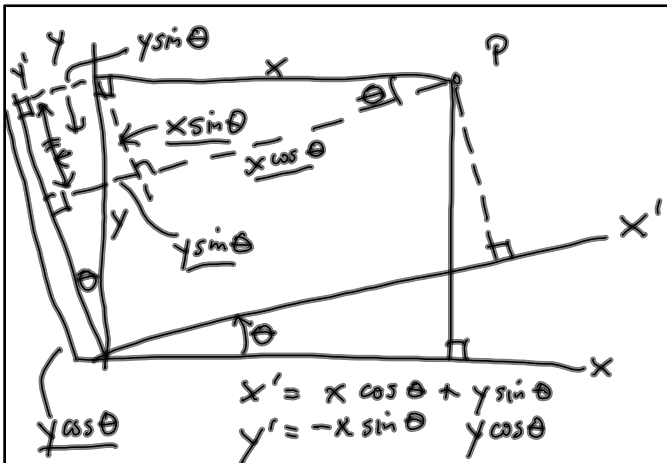

$x' = x\cos\theta + y\sin\theta$

$y' = -x\sin\theta \quad y\cos\theta$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

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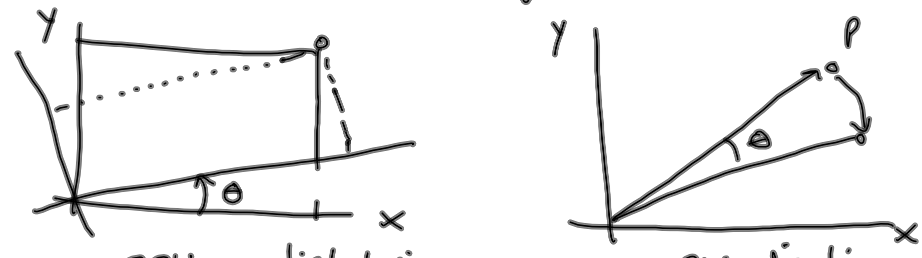
Can interpret rotation in 2 ways



CCW or anticlockwise CW direction

2D rotation matrix : orthogonal matrix

- determinant = 1 $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

  $\cos^2\theta + \sin^2\theta = 1$

- inverse = transpose

  $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

reflection matrix det( ) = −1

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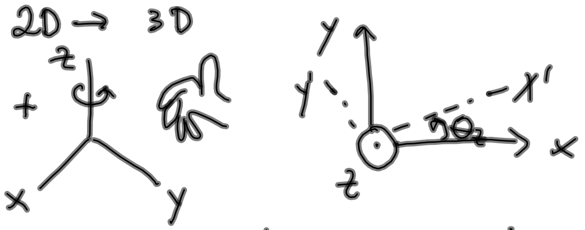
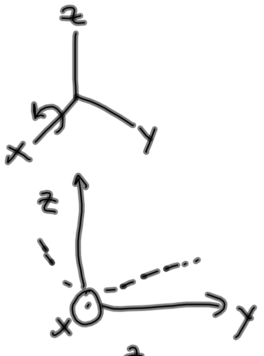
2D → 3D



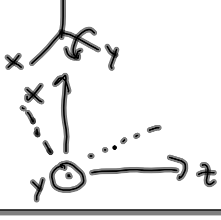
$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

M_z, R_3



$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

M_x, R_1



$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

M_y, R_2

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$M = M_z \, M_y \, M_x$

any rotation $\quad R_3 \;\; R_2 \;\; R_1$

elementary rotation matrices

euler angles $\quad M_x M_y \neq M_y M_x$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ 1 parameter transf.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \lambda \cdot \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ 2 param transf.

$a = \lambda \cos\theta$

$b = \lambda \sin\theta$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

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