Continue 2D range problem 14-1

$\Delta = (B^T W B)^{-1} B^T W f = \begin{bmatrix} .0672 \\ .0925 \end{bmatrix}$

$\begin{pmatrix} x^0 \\ y^0 \end{pmatrix} = \begin{pmatrix} x^0 \\ y^0 \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix}$

recompute all entries in B, f

$\Delta_2 = \begin{bmatrix} -.0004 \\ .0013 \end{bmatrix}$

$\Delta_3 = \begin{bmatrix} 1.16e-6 \\ 1.34e-5 \end{bmatrix}$

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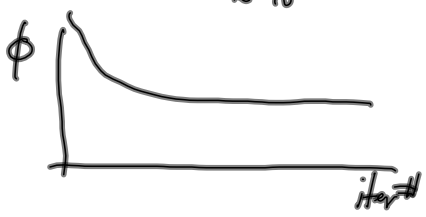
Phi_prev = V'*W*V   (last iteration)
Phi_curr = V'*W*V   (current iteration)
if( abs((Phi_curr - Phi_prev)/Phi_curr) < threshold )
    keep_going = 0
    disp('we have converged')
end
if(niter > 10)
    keep_going = 0
    disp('we did not converge')
end
```

threshold $\approx 10^{-8}$
$\approx 10^{-10}$



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7-parameter problem 14-2

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \cdot M \left(\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \right)$$

$x = \lambda M_1 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + t_x$ $\quad M_i = i^{th}$ row of M

(l.c.) $y = \lambda M_2 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + t_y$ $\quad R_1, R_2, R_3$

$z = \lambda M_3 \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + t_z$

(u.c.)

$F \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \lambda M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} = 0, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\partial F / \partial \lambda = -M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$, $\frac{\partial F}{\partial t_x} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$, $\frac{\partial F}{\partial t_y} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$, $\frac{\partial F}{\partial t_z} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

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14-3

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \lambda M \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix}$$

$$\frac{\partial F}{\partial w} = -\lambda \frac{\partial M}{\partial w} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$M_w = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos w & \sin w \\ 0 & -\sin w & \cos w \end{bmatrix}$$

$$M = M_K M_\varphi M_w$$

$$\Rightarrow \frac{\partial M}{\partial w} = M_K M_\varphi \frac{\partial M_w}{\partial w}$$

$$\frac{\partial M_w}{\partial w} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\sin w & \cos w \\ 0 & -\cos w & -\sin w \end{bmatrix}$$

$$\frac{\partial F}{\partial \varphi} = -\lambda \frac{\partial M}{\partial \varphi} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\frac{\partial M}{\partial \varphi} = M_K \frac{\partial M_\varphi}{\partial \varphi} M_w$$

$$M_\varphi = \begin{bmatrix} \cos\varphi & 0 & -\sin\varphi \\ 0 & 1 & 0 \\ \sin\varphi & 0 & \cos\varphi \end{bmatrix}$$

$$\frac{\partial M_\varphi}{\partial \varphi} = \begin{bmatrix} -\sin\varphi & 0 & -\cos\varphi \\ 0 & 0 & 0 \\ \cos\varphi & 0 & -\sin\varphi \end{bmatrix}$$

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14-4

$$\frac{\partial F}{\partial k} = -\lambda \frac{\partial M}{\partial k} \begin{pmatrix} x \\ Y \\ z \end{pmatrix}$$

$$\frac{\partial M}{\partial k} = \frac{\partial M_k}{\partial k} \cdot M_\varphi M_w$$

$$M_K = \begin{bmatrix} \cos k & \sin k & 0 \\ -\sin k & \cos k & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial M_k}{\partial k} = \begin{bmatrix} -\sin k & \cos k & 0 \\ -\cos k & -\sin k & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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