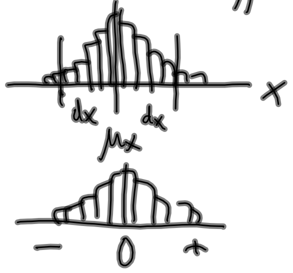
Lecture 22 — Suggestions for HW4 #1 — 22-1



1. convert to zero mean
   $X0 = X - \bar{X}$
2. abs. value folding left half onto right half
   $aX0 = abs(X0)$
3. $sX0 = sort(aX0)$
4. $dx = sX0(4500)$

check $std(X0) * 1.645$

Oct 16-4:27 PM

22-2

$\boxed{\text{model}}$

$\left.\begin{matrix} l \\ \Sigma_{ll} \end{matrix}\right\} \to \boxed{LS} \to$ x, Δ, v, $\hat{l}$ — estimation

$\left.\begin{matrix} \Sigma_{xx} \\ \Sigma_{\Delta\Delta} \\ \Sigma_{vv} \\ \Sigma_{\hat{l}\hat{l}} \end{matrix}\right\} \to$ confidence intervals, confidence region, confidence volume, conf. hypervolume, ⋮, hypothesis test, blunder detection

error propagation

l

$W_{ll} \to Q_{ll}$

Q_{xx}
$Q_{\Delta\Delta}$
Q_{vv}
$Q_{\hat{l}\hat{l}}$

Oct 16-4:27 PM

Remember Σ describes precision, random errors 22-3
(Q)

Σ does not describe biases, syst. errors
gross errors, blunders, ...

$$\Sigma_{ll} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & & \sigma_n^2 \end{bmatrix}$$

independent
uncorrelated
Σ diagonal

$$\Sigma_{ll}^{-1} = \begin{bmatrix} 1/\sigma_1^2 & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2^2} & & \vdots \\ \vdots & & \ddots & \frac{1}{\sigma_n^2} \\ 0 & \cdots & & \end{bmatrix}, \quad \underbrace{\sigma_0^2 \Sigma_{ll}^{-1}}_{= W_{ll}} = \begin{bmatrix} \sigma_0^2/\sigma_1^2 & & 0 \\ & \sigma_0^2/\sigma_2^2 & \\ & & \ddots \\ 0 & & \sigma_0^2/\sigma_n^2 \end{bmatrix}$$

Oct 16-4:27 PM

22-4

$\sigma_0^2 \Sigma_{ll}^{-1} = W_{ll}$

$\frac{1}{\sigma_0^2} \Sigma_{ll} = Q_{ll}$

$\Sigma_{ll} = \sigma_0^2 Q_{ll}$

$\boxed{\Sigma = \sigma_0^2 Q}$ can do E.P. with either Σ or Q

Oct 16-4:27 PM

General Law of Error Prop. 22-5

$\vec{y} = A\vec{x}$, know Σ_{xx}, $\Sigma_{yy} = ?$

$E\{\vec{y}\} = E\{A\vec{x}\} = A \cdot E\{x\}$

$\mu_y = A \cdot \mu_x$

$\Sigma_{yy} = E\{(y-\mu_y)(y-\mu_y)^T\}$

$= E\{(Ax - A\mu_x)(Ax - A\mu_x)^T\}$

$(x^T A^T - \mu_x^T A^T)$

$= E\{A(x-\mu_x)(x^T - \mu_x^T)A^T\}$

$= A\,E\{(x-\mu_x)(x^T - \mu_x^T)\}A^T$

$= A\,E\{(x-\mu_x)(x-\mu_x)^T\}A^T$

Oct 16-4:27 PM

$\boxed{\Sigma_{yy} = A\,\Sigma_{xx}A^T}$ general error propagation law 22-6

$y = Ax$

$y = Ax + b$ same result

shown case where y is linear function of x

Oct 16-4:27 PM

$y = Ax$ lin.

22-7

$\vec{y} = F(\vec{x})$

$y_1 = f_1(x_1, x_2 \ldots x_n)$

$y_2 = f_2(x_1, x_2 \ldots x_n)$

$\vdots$

$y_m = f_m(x_1, x_2 \ldots x_n)$

$\vec{y} \approx F(x^0) + \left[\frac{\partial F}{\partial x}\right] \cdot \Delta x$

$\approx F(x^0) + J \cdot \Delta x$

$\vec{y} \approx J \cdot \Delta x + F(x^0)$

$\boxed{\vec{y} = A \cdot x + b}$ from before

for nonlinear case $\Sigma_{yy} = J \Sigma_{xx} J^T$

linear $\Sigma_{yy} = A \Sigma_{xx} A^T$

Oct 16-4:27 PM

examples

22-8

$y = a_1 x_1 + a_2 x_2 \qquad \Sigma_{\binom{x_1}{x_2}} = \begin{bmatrix} \sigma_{x_1}^2 & 0 \\ 0 & \sigma_{x_2}^2 \end{bmatrix}$

Oct 16-4:27 PM