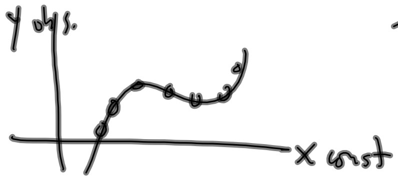
Session 27 exam 1 : mean : 83 27-1



$n = 7$

$n_0 = 4$

$r = 3$

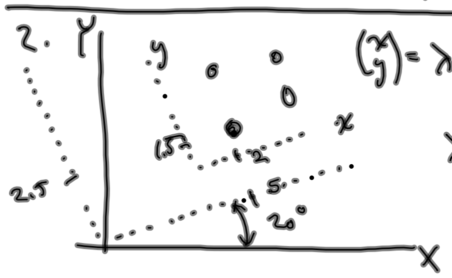
$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$ linear

$v + B\Delta = f$

$y + v_y - a_0 - a_1 x - a_2 x^2 - a_3 x^3 = 0$

$$v_y + \begin{bmatrix} -1 & -x & -x^2 & -x^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = -y$$

2.



$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

$n = 8$

$n_0 = 3$

$r = 5$

$\lambda = 1.0000$

XY const $t_x \approx -3$

xy obs $t_y \approx -1$

$V + B\Delta = f$ condition equations 27-2

(9,3)

LS

$(B^T W B)\Delta = B^T W f$ normal equations

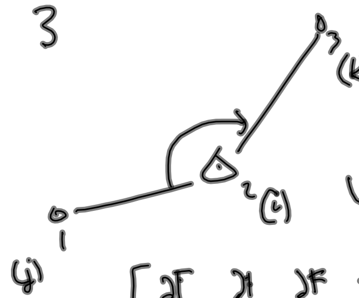
$N \Delta = t$

3×3

(d) substitution to make linear $\cos\theta = a$, $\sin\theta = b$

3

27-3



$F_\theta = \theta_{jk} - \theta_{computed}$

(obs)

$V + B\delta = f$

$$\left[\frac{\partial F}{\partial x_1} \quad \frac{\partial F}{\partial y_1} \quad \frac{\partial F}{\partial x_3} \quad \frac{\partial F}{\partial y_3}\right]$$

$$\frac{\partial F}{\partial x_j} \quad \frac{\partial F}{\partial y_j} \quad \frac{\partial F}{\partial x_k} \quad \frac{\partial F}{\partial y_k}$$

$$\left[\frac{\Delta y_{ij}}{D_{ij}^2} \quad \frac{-\Delta x_{ij}}{D_{ij}^2} \quad \frac{-\Delta y_{ik}}{D_{ik}^2} \quad \frac{\Delta x_{ik}}{D_{ik}^2}\right]$$

$$\left[\frac{-3}{34} \quad \frac{5}{34} \quad \frac{-6}{52} \quad \frac{4}{52}\right]$$

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$F = \theta_{jk} - [az_{ik} - az_{ij}] = 0$

27-4

$(155°)$

$$\left(\operatorname{atan}\left(\frac{4}{6}\right) - \left[\operatorname{atan}\left(\frac{-5}{-3}\right) + 180°\right]\right)$$

$33.69°$ $\quad$ $59.03° - 180$

$F = 155 - 154.65 = .35$

$f = -F = -.35°$ or $-.006\ R$

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E.P. $\Delta = (B^TWB)^{-1}B^TWf$ Ind Obs 27-5

$Q_{\Delta\Delta}$?

$\Delta = [\;]f$

$f = [\;]l$

$f = d - l$

$f = (-I)l + d$

$y = Ax + b$

$Q_{ff} = (-I)Q_{ll}(-I^T) = Q_{ll}$

Q, W w/o subscripts = Q_{ll}, W_{ll}

$\Delta = (B^TWB)^{-1}B^TWf$ $y = Ax$

$$Q_{\Delta\Delta} = (B^TWB)^{-1}\underbrace{B^TW\cdot \underbrace{QW}_{I}B(B^TWB)^{-1}}_{I}$$

$$Q_{\Delta\Delta} = (B^TWB)^{-1} = N^{-1}$$

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$\Sigma_{\Delta\Delta} = \sigma_0^2 Q_{\Delta\Delta}$ if you chose $\sigma_0^2 = 1$ 27-6

assumption: errors in obs are normally distr.

normal distr.

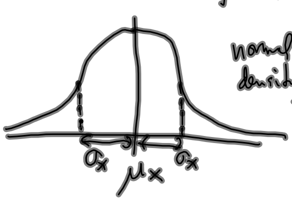
Multi-variate normal MVN

t, χ^2 (chi-squared), F

normal distr. $f(x) = \dfrac{1}{\sigma_x\sqrt{2\pi}}\exp -\frac{1}{2}\dfrac{(x-\mu_x)^2}{\sigma_x^2}$

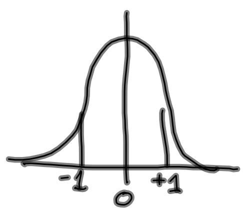
$z = \dfrac{x-\mu_x}{\sigma_x}$ $\mu_z = 0,\ \sigma_z = 1$

$f(z) = \dfrac{1}{\sqrt{2\pi}}\exp -\frac{1}{2}z^2$



normal density f.

std. normal dens. f.



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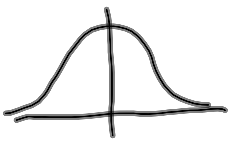
MVN density 27-7

$$f(\vec{x}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} \exp -\tfrac{1}{2}\left((\vec{x}-\vec{\mu}_x)^T \Sigma_{xx}^{-1} (\vec{x}-\vec{\mu}_x)\right)$$

n: dim of $\vec{x}$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad \vec{\mu}_x = \begin{bmatrix} \mu_{x_1} \\ \mu_{x_2} \\ \vdots \\ \mu_{x_n} \end{bmatrix} \quad \Sigma_{xx} = \begin{bmatrix} \sigma_{x_1}^2 & \sigma_{x_1x_2} & & \sigma_{x_1x_n} \\ & \sigma_{x_2}^2 & & \vdots \\ & & \ddots & \\ & & & \sigma_{x_n}^2 \end{bmatrix}$$



n=1

n=3, 4, ... cannot draw picture of density function

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χ^2 chi-squared 27-8

$x_1, x_2, \dots, x_n$ independent, N(0,1)

then

$$x_1^2 + x_2^2 + \dots + x_n^2 \sim \chi_n^2$$

n: degrees of freedom

$u: \chi_n^2$

$$f(u) = C_n u^{(n-2)/2} e^{-u/2}$$

$$C_n = \frac{1}{2^{n/2}\Gamma(n/2)}$$

matlab function Y = gamma(x)

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