## Session 28

28-1

$$\Gamma(n) = \int_0^\infty e^{-t} t^{n-1}\, dt$$

matlab
y = gamma(x)

n: integer, then

$\Gamma(n+1) = n\,\Gamma(n)$, $\Gamma(1) = 1$ } $\Gamma(n) = (n-1)!$, $\Gamma(n+1) = n!$

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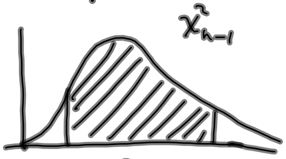
n samples of RV

$\bar{X} = \frac{\sum x_i}{n}$, $s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$

$\sigma^2$ population value of variance of X

$\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$




should fall in "middle" of density function

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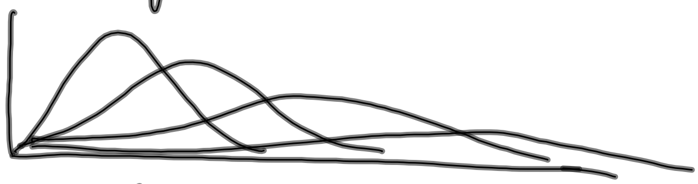
for n large

28-2



$\frac{(n-1)\cdot s^2}{\sigma^2} \sim \chi^2_{n-1}$ $s^2 = \frac{\sum(x-\bar{x})^2}{n-1}$

$\frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi^2_r$, $\hat{\sigma}_0^2 = \frac{v^T W v}{r}$

will compare $\hat{\sigma}_0^2$ with $\sigma_0^2$

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Normal (gaussian), MVN, t, $\chi^2$, F 28-3

$$z = \frac{\hat{x} - \mu_x}{\sigma_{\hat{x}}}$$

$$t = \frac{\hat{x} - \mu_x}{s_{\hat{x}}}$$

$\bar{x}$, $s^2$, $\frac{\bar{x} - \mu}{s/\sqrt{n}} \sim t_n$

$f_n(t)$ mistake in OLS

$$= \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{n\pi}\,\Gamma(n/2)} \left(1 + \frac{t^2}{n}\right)^{-\frac{n+1}{2}}$$

as $n \to \infty$, $t \to z$

large $t \to z\,[N(0,1)]$

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F distr. 2 $\chi^2$: $\chi^2_m$, $\chi^2_n$

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Then 28-4

$$\mu = \frac{\chi^2_m / m}{\chi^2_n / n} \sim F_{m,n}$$

Common use of F compare 2 variance estimates

$$\frac{(n_1-1)\, s_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1}, \quad \frac{(n_2-1)\, s_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$$

$$\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{n_1-1,\, n_2-1}$$

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$$F_{m,n}(\mu) = \frac{\Gamma\left(\frac{m+n}{2}\right)}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)} \cdot \left(\frac{m}{n}\right)^{m/2} \frac{\mu^{(m-2)/2}}{\left[1 + \left(\frac{m}{n}\right)\mu\right]^{\frac{m+n}{2}}}$$

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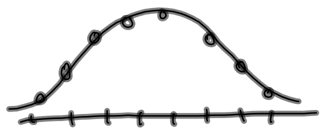
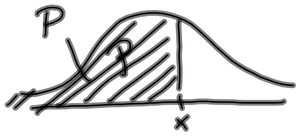
to get numerical values pdf, cdf, icdf 28-5

y = pdf('norm', μ, σ)



P = cdf('norm', x, μ, σ)




x = icdf('norm', P, μ, σ)

'norm', 'chi2', 't', 'f'
n, n, $n_1, n_2$

MVN ??

get numerical value from matlab
other methods: tables
sci calculators

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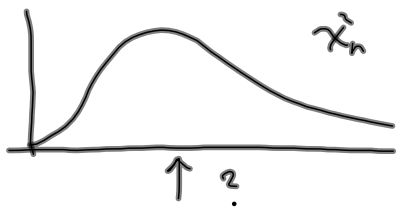
$\sigma_0^2$ choose prior 28-6
a priori

$\hat{\sigma}_0^2$ post adjustment estimate of $\sigma_0^2$
a posteriori

$$\hat{\sigma}_0^2 = \frac{v^T W v}{r}, \quad \underbrace{\frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} \sim \chi_r^2}_{\text{global test}}$$

$$\frac{r \cdot \frac{v^T W v}{r}}{\sigma_0^2} = \frac{v^T W v}{\sigma_0^2} \sim \chi_r^2$$

test statistic $\frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2}$ or $\frac{v^T W v}{\sigma_0^2}$

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