Lecture 30 30-1

Global Test: 2-sided α: L.O.S. $(1-\beta)$

if $cv_1 < \chi^{2*} < cv_2$ then accept H_0

otherwise reject H_0

$cv_1 = icdf('chi2', \alpha/2, r)$

$cv_2 = icdf('chi2', 1-\alpha/2, r)$

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1 sided test $\boxed{\chi^{2*} = \frac{v^T W v}{\sigma_0^2}}$

if $\chi^{2*} < cv$ then accept H_0

otherwise reject H_0

$cv = icdf('chi2', 1-\alpha, r)$

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another way to state 2-sided G.T. 30-2

if $(\alpha/2 < P_{\chi^{2*}} < 1-\alpha/2)$ then accept H_0

... (cv_1) χ^{2*} (cv_2)

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identities 30-3

1. $v \to \infty$
   $\chi^2_v \to$ normal
2. $v_2 \to \infty$, $F_{v_1,v_2} \to \chi^2_{v_1}/v_1$, $F_{v_1,\infty} = \chi^2_{v_1}/v_1$
3. $v \to \infty$
   $t_v \to$ std. normal
4. $t_v^2 = F_{1,v}$
5. $F_{1,\infty} = t_\infty^2 = z^2 = \chi^2_1$
6. $x \sim F_{r,\infty} \leftrightarrow r \cdot x \sim \chi^2_r$
   $x \sim \frac{1}{r}\chi^2_r$

$x \sim \chi^2_r \leftrightarrow \frac{x}{r} \sim F_{r,\infty} \leftarrow$

$\chi^{2*}_r = \boxed{\frac{v^T W v}{\sigma_0^2} \sim \chi^2_r}$ chi-squared test

$\frac{\frac{v^T W v}{\sigma_0^2}}{r} \sim F_{r,\infty}$, $\frac{\frac{v^T W v}{r}}{\sigma_0^2} \sim F_{r,\infty}$

$\boxed{\hat{\sigma}_0^2/\sigma_0^2 \sim F_{r,\infty}}$ F test

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Consequences of outcome of G.T. ? 30-4

pass G.T. $\Sigma_{xx} = \sigma_0^2 Q_{xx}$ prior assumed value

not pass G.T. $\Sigma_{xx} = \hat{\sigma}_0^2 Q_{xx}$

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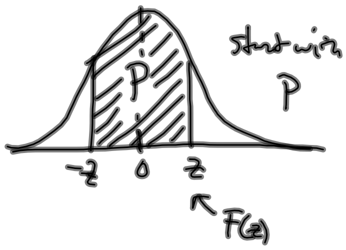
Confidence intervals

30-5

$\frac{\hat{x}-\mu}{\sigma_x} \sim z$



$F(z) = \frac{1}{2} + \frac{P}{2} = \frac{1+P}{2}$

$z = \text{icdf}\left('norm', \frac{P+1}{2}, 0, 1\right)$

$\text{Prob}\left(-z < \frac{\hat{x}-\mu}{\sigma_x} < +z\right) = P$ ← choose

$(-z\sigma_x < \hat{x}-\mu < z\sigma_x)$

$(-\hat{x} - z\sigma_x < -\mu < -\hat{x} + z\sigma_x)$

$(\hat{x} + z\sigma_x > \mu > \hat{x} - z\sigma_x)$

$\text{Prob}\left(\hat{x} - z\sigma_x < \mu < \hat{x} + z\sigma_x\right) = P$

centered at $\hat{x} \pm z\sigma_x$

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30-6

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \end{bmatrix} \leftarrow \begin{bmatrix} \sigma_{x_1}^2 & \cdot & \cdot & \cdot & & \\ \cdot & \sigma_{y_1}^2 & \cdot & \cdot & & \\ \cdot & \cdot & \sigma_{z_1}^2 & \cdot & & \\ \cdot & \cdot & \cdot & \sigma_{x_2}^2 & & \\ & & & & \sigma_{y_2}^2 & \\ & & & & & \sigma_{z_2}^2 \end{bmatrix}$$

$(B^T W B)^{-1} = N^{-1} = Q_{\Delta\Delta}$, $\sigma_0^2 Q_{\Delta\Delta} = \underline{\underline{\Sigma_{\Delta\Delta}}}$

pass G.T.

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if do not pass G.T., $\Sigma_{\Delta\Delta} = \hat{\sigma}_0^2 Q_{\Delta\Delta}$

$\frac{x-\mu_x}{\sigma_x} \sim z \Rightarrow \underline{\underline{\frac{x-\mu_x}{S_x} \sim t_r}}$

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