Lecture 34

34-1

for conf. regions need eigenvalues + eigenvectors

3x3 + matlab calculator   [V,D] = eig(S)

2x2 — by hand

solve $Av = \lambda v$, $A = \Sigma$

$\det(A - \lambda I) = 0$ (2x2)

$$\begin{vmatrix} \sigma_x^2 - \lambda & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 - \lambda \end{vmatrix} = 0$$ yields quadratic eqn.

$$\lambda = \frac{\sigma_x^2 + \sigma_y^2}{2} \pm \left[\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2 \right]^{1/2}$$

get orientation of ellipse:

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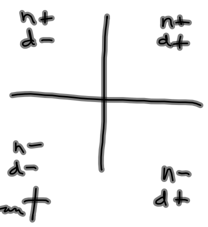
34-2

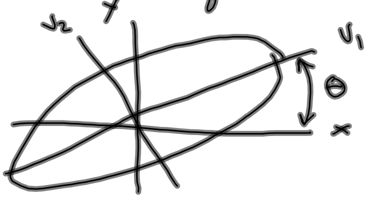
① $A \Sigma A^T = D$

$\vdots$

$$2\theta = \operatorname{atan}\left(\frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} \right)$$



get 2θ into correct quadrant



② have λ, $Av = \lambda v$, $\begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \lambda \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$a_{11} v_1 + a_{12} v_2 = \lambda v_1$

$(a_{11} - \lambda) v_1 = -a_{12} v_2$

$$\left. v_1 = \frac{-a_{12}}{a_{11} - \lambda} \cdot v_2 \right\} \quad v_1 = \frac{-\sigma_{xy}}{\sigma_x^2 - \lambda} \cdot v_2$$

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Confidence Circles

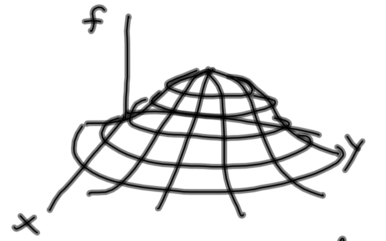
$R = \text{cep2}(P, cov)$ 34-3

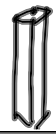
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}, \quad \Sigma_{xx} = \begin{bmatrix} \odot & \cdots & \odot & \cdots \\ \cdots & & \cdots & \\ \odot & \cdots & \odot & \cdots \\ \cdots & & \cdots & \end{bmatrix}; \quad \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \\ x_n \\ y_n \end{bmatrix} \quad \Sigma_{xx} = \begin{bmatrix} [\cdot\ \cdot] & \cdots & \\ \cdots & [\cdot\ \cdot] & \\ & & \ddots \end{bmatrix}$$

MVN $(n=2)$

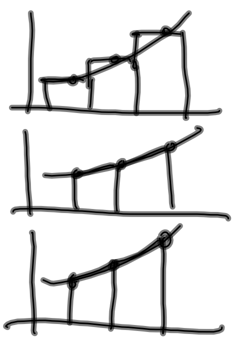
$$f(\vec{x}) = \frac{1}{2\pi^{n/2}\sqrt{|\Sigma|}} \exp^{-\frac{1}{2}\left[(x-\mu)^T \Sigma^{-1} (x-\mu)\right]}$$

for $n=2$, bivariate normal density function





2D



1D

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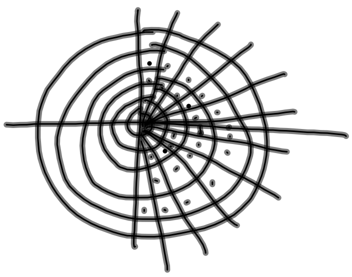

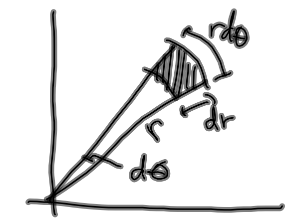

area of base of vol element

$r\,d\theta\,dr$

$$\sum_r \sum_\theta f(r,\theta) \cdot r\,d\theta\,dr$$

$$\iint_{\text{circle}} f(x,y)\,dx\,dy$$

change of variable $xy \to r, \theta$

$x = r\cos\theta$

$y = r\sin\theta$

$$\int_0^{2\pi}\int_0^{r} f(r\cos\theta, r\sin\theta)\,\underbrace{|J|}_{r}\,dr\,d\theta \to \text{same formula as by intuition}$$

$$J = \begin{bmatrix} \partial x/\partial r & \partial x/\partial\theta \\ \partial y/\partial r & \partial y/\partial\theta \end{bmatrix}, \quad |J| = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$$

z vs t 34-5

χ^2 vs F

MVN vs MVT *

I/O $Q_{\Delta\Delta}$, $Q_{\hat{\ell}\hat{\ell}}$, Q_{vv}

$y = Ax + b$, $\Sigma_{yy} = A\Sigma_{xx}A^T$

$\hat{\ell} = \ell + v$, $v + B\Delta = f$, $v = f - B\Delta$

$\hat{\ell} = \ell + f - B\Delta$, $f = d - \ell$

$\hat{\ell} = \ell + d - \ell - B\Delta = -B\Delta + d$

$\hat{\ell} = -B\Delta + d$, $Q_{\hat{\ell}\hat{\ell}} = -B\,Q_{\Delta\Delta}\,-B^T$

$$\boxed{Q_{\hat{\ell}\hat{\ell}} = BN^{-1}B^T}$$

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$Q_{vv} = ?$ $y = Ax + b$ 34-6

$v = f - B\Delta$

$v = f - BN^{-1}t$

$v = d - \ell - BN^{-1}B^TW(d - \ell)$

$v = \cdots$

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