Lecture 35

I/o $Q_{\hat{l}\hat{l}} = BN^{-1}B^T$ 35-1

$v = f - B\delta$ $\quad Q_{vv} = ?$

$f - BN^{-1}t$

$f - BN^{-1}B^TWf$, $\quad v = (I - BN^{-1}B^TW)f$

$Q_{ff} = Q = Q_{ll}$

$Q_{vv} = (I - BN^{-1}B^TW)Q(I - BN^{-1}B^TW)^T$

$(Q - BN^{-1}B^T)(I - WBN^{-1}B^T)$

$= Q + BN^{-1}B^TWBN^{-1}B^T - BN^{-1}B^T - BN^{-1}B^T$

$= Q + BN^{-1}B^T - 2BN^{-1}B^T$

$\boxed{Q_{vv} = Q - BN^{-1}B^T}$

$Q_{vv} = Q - Q_{\hat{l}\hat{l}}$

$\boxed{Q = Q_w + Q_{\hat{l}\hat{l}}}$

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obs only $Q_{..} = ?$ 35-2

$Av = f$, $k = W_e f$, $v = QA^Tk$

1. $Q_w = QA^TW_eAQ$
2. $Q_{\hat{l}\hat{l}} = Q - Q_{vv}$

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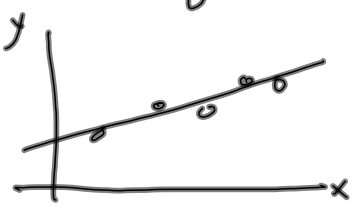
General LS (mixed model) 35-3

$$Av + B\Delta = f$$

I/O $A = I \Rightarrow v + B\Delta = f$

o/o $B = 0 \Rightarrow Av = f$

n_0: # observations to reconstruct problem including location of all observations



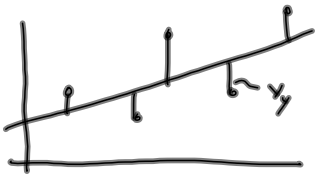
$x \& y$ are observed

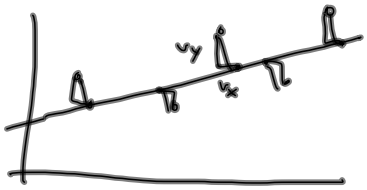
num pts. $= 5$

$n = 10$

$n_0 = 4 + 3 = 7$

$r = 3$





$u = 2$

$c = r + u = 5$

$y = mx + b$

u: number of parameters

$F: y_i - mx_i - b = 0$

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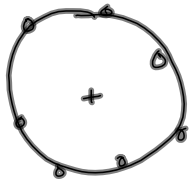
35-4



fit 7 pts to circle, obs. $x \& y$

$n = 14$

$n_0 = 6 + 4 = 10$ (8, 5, 9, 6, 10, 7)

parameters $\rightarrow X_c, Y_c, R$

$3 + 7 = 10$

$r = 4$

$R = \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}$

$F = R - [(x_i - x_j)^2 + (y_i - y_j)^2]^{1/2} = 0$

$c = r + u = 4 + 3 = 7$ ✓

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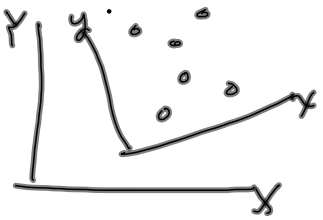
Coord transformation: 35-5



pts = 6

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \lambda \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

↑ obs. ↑ obs.

par $\lambda, \theta, t_x, t_y$ (4)

$n = 24$

$n_0 = 8 + 4 \times 2 = 16$

$r = 8$

$C = r + n$ $8 + 4 = 12$ ✓ 2 eqn's per point

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back to simple line fit problem 35-6

y obs.
x const.

$n = 3$
$n_0 = 2$
$r = 1$

I/O $u = 2$, $C = 3$, $c = n$ $(= r + n)$

O/O $\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2}$ $c = r$, $= 1$

GLS $y_1 = mx_1 + b$ $\Rightarrow$ $b = y_1 - mx_1$

$y_2 = mx_2 + b$
$y_3 = mx_3 + b$

$y_2 = mx_2 + y_1 - mx_1$
$y_3 = mx_3 + y_1 - mx_1$

$y_2 - y_1 = m(x_2 - x_1)$
$y_3 - y_1 = m(x_3 - x_1)$

if solve with 1 par $\Rightarrow$ GLS

$n = 3$
$n_0 = 2$
$r = 1$

$u = 1$, $C = r + n = 2$

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