Lecture 36 36-1

in remaining 9 sessions: GLS, par. constraints, Sequential LS, blunder detection (red.#), wrap up

GLS $Av + B\delta = f$

I/O $A = I$, $v + B\delta = f$

O/O $B = 0$, $Av = f$

GLS typical problems

curve fit, surface, coord. transf. where all coordinates are observed.

if model linear: $A\hat{\ell} + Bx = d$

$$A(\ell + v) + Bx = d$$

$$Av + Bx = \underbrace{d - A\ell}_{f}$$

$$\boxed{Av + Bx = f} \qquad \boxed{Av + B\delta = f}$$

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assume non linear problem 36-2

$$F(\ell, x) \approx F(\ell^0, x^0) + \underbrace{\frac{\partial F}{\partial \ell}}_{A} \delta\ell + \underbrace{\frac{\partial F}{\partial x}}_{B} \delta x = 0$$

$\ell + v = \ell^0 + \delta\ell$

$\delta\ell = (\ell - \ell^0) + v$

$$Av + B\delta = \underbrace{-F(\ell^0, x^0) - A(\ell - \ell^0)}_{f}$$

$$Av + B\delta = f$$

practical solution hints: 2 vectors: ℓ original, ℓ^0 updated

after counting,

$c = r + u$

$$\underset{c,n}{A}\;\underset{n,1}{v} + \underset{c,u}{B}\;\underset{u,1}{\delta} = \underset{c,1}{f}$$

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inside iteration loop 36-3

$A = \text{zeros}(c,n)$
$B = \text{zeros}(c,u)$
$f = \text{zeros}(c,1)$

$$\Phi' = v^T W v - 2k^T(Av + B\Delta - f)$$
$$- 2(Av + B\Delta - f)^T k$$

$$\frac{\partial \phi'}{\partial v} = \cancel{2}v^T W - \cancel{2}k^T A = 0 \quad \text{(row vector)}$$

$$\frac{\partial \phi'}{\partial \Delta} = -\cancel{2}k^T B = 0 \quad ''$$

$$\frac{\partial \phi'}{\partial k} = -\cancel{2}(Av + B\Delta - f)^T = 0 \quad ''$$

$$Wv - A^T k = 0 \quad \text{(col vector)} \Big\} \; -Wv + A^T k = 0$$
$$-B^T k = 0 \quad '' \qquad Av + B\Delta = f$$
$$-(Av + B\Delta - f) = 0 \quad '' \qquad B^T k = 0$$

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$$-Wv + A^T k = 0$$
$$Av + B\Delta = f$$
$$B^T k = 0$$

choose $\begin{pmatrix} v \\ k \\ \Delta \end{pmatrix}$

$$\begin{array}{c c} & \begin{array}{ccc} n & c & u \end{array} \\ \begin{array}{c} n \\ c \\ u \end{array} & \begin{bmatrix} -W & A^T & 0 \\ A & 0 & B \\ 0 & B^T & 0 \end{bmatrix} \end{array} \begin{bmatrix} v \\ k \\ \Delta \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \end{bmatrix}$$

full normal equations

$(n+c+u \times n+c+u)$

elimination
$$-Wv + A^T k = 0$$
$$Av + B\Delta = f$$
$$B^T k = 0$$

$$Wv = A^T k$$
$$v = QA^T k$$
$$AQA^T k + B\Delta = f$$
$$k = (AQA^T)^{-1}(f - B\Delta)$$
$$B^T W_e (f - B\Delta) = 0 \Rightarrow B^T W_e B\Delta = B^T W_e f$$

$$AQA^T = Q_e$$
$$Q_e^{-1} = W_e$$

solve numerically
$$\Delta = (B^T W_e B)^{-1} B^T W_e f$$
$$\Delta = N^{-1} t$$
$$k = W_e(f - B\Delta)$$
$$v = QA^T k$$

back substitution

solve δ, k, v 36-5

$x^{0}_{new} = x^{0}_{old} + \delta$

$l^{0}_{new} = l_{original} + v \longleftarrow l^{0}_{prev} = l^{0}_{new}$

$\Delta l = l^{0}_{new} - l^{0}_{previous}$

E.P.

$Q_{\delta\delta} = N^{-1} \quad N = B^{T} W_e B$

$Q_{vv} = QA^{T} W_e A Q - QA^{T} W_e B N^{-1} B^{T} W_e A Q$

$Q_{\hat{\ell}\hat{\ell}} = Q - Q_{vv}, \quad Q_{\ell\ell} = Q$

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