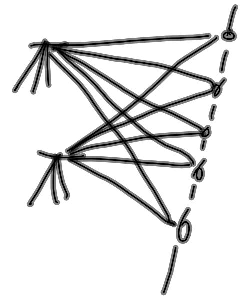
Lecture 38 Constraints w/ obs. only method 38-1



?

conventional way

$n = 10$

$n_0 = 4+3 = 7$

$r = 3$

$u = 10 \quad x_1, y_1 \ldots x_5 y_5$

$s = 3$

$c = 10$

$c + s = r + u$

$10 + 3 = 3 + 10$ ✓

obs. only range obs 10, coord obs 10 } 20

$n = 20$

$n_0 = 7$

$r = 13$

10 distance/range equations, 3 line equations } cond eq's 13

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weights for x,y's very small

weights for range eq's

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obs only $r = 13$, conv. $r = 3$

$$\boxed{\begin{array}{l} Av + B\Delta = f \\ C\Delta = g \end{array}}$$

Conventional constraint method

$$\phi' = v^T W v - 2k^T(Av + B\Delta - f) - 2k_c^T(C\Delta - g)$$

$$- 2(Av + B\Delta - f)^T k - 2(C\Delta - g)^T k_c$$

$$\left.\begin{array}{l} \partial\phi'/\partial v = \cancel{2}v^T W - \cancel{2}k^T A = 0 \\ \partial\phi'/\partial k = -\cancel{2}(Av + B\Delta - f)^T = 0 \\ \partial\phi'/\partial\Delta = -\cancel{2}k^T B - \cancel{2}k_c^T C = 0 \\ \partial\phi'/\partial k_c = -\cancel{2}(C\Delta - g)^T = 0 \end{array}\right\} \begin{array}{l} Wv - A^T k = 0 \\ -(Av + B\Delta - f) = 0 \\ -B^T k - C^T k_c = 0 \\ -(C\Delta - g) = 0 \end{array} \left\{ \begin{array}{l} -Wv + A^T k = 0 \\ Av + B\Delta = f \\ B^T k + C^T k_c = 0 \\ C\Delta = g \end{array}\right.$$

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$-Wv + A^T k = 0$
$Av + B\delta = f$
$B^T k + C^T k_c = 0$
$C\delta = g$

$$\begin{bmatrix} -W & A^T & 0 & 0 \\ A & 0 & B & 0 \\ 0 & B^T & 0 & C^T \\ 0 & 0 & C & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \delta \\ k_c \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \\ g \end{bmatrix}$$

full normal equations

order n, c, u, s $\quad n+c+u+s$

elimination: $Wv = A^T k$, $v = QA^T k$

$\underbrace{AQA^T}_{Q_e} k + B\delta = f$, $\quad k = W_e(f - B\delta)$

$B^T W_e (f - B\delta) + C^T k_c = 0$

$-B^T W_e B\delta + C^T k_c = -B^T W_e f$

if problem solvable w/o constraints, then N nonsingular

$-N\delta = -t - C^T k_c \leftarrow$

$\delta = \underbrace{N^{-1} t}_{\delta^0} + \underbrace{N^{-1} C^T k_c}_{\delta\delta}$

$C[N^{-1}t + N^{-1}C^T k_c] = g$, $\quad CN^{-1}C^T k_c = g - CN^{-1}t$, $\quad k_c = (CN^{-1}C^T)^{-1}(g - CN^{-1}t)$

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$-N\delta + C^T k_c = -t$
$C\delta = g$

$$\begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \delta \\ k_c \end{bmatrix} = \begin{pmatrix} -t \\ g \end{pmatrix}$$

solve for case N singular

$$\begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \alpha & \beta^T \\ \beta & \gamma \end{bmatrix} \quad \alpha \text{ sym}, \gamma \text{ sym}$$

$Q_{\delta\delta} = -\alpha \quad$ (N singular)

$Q_{\delta\delta} = N^{-1}(I - C^T(CN^{-1}C^T)^{-1}CN^{-1}) \quad$ (N full rank)

$\Sigma_{\delta\delta} = \sigma_0^2 Q_{\delta\delta} \quad \ldots$

Note the inverse applied to the quantity C*N^-1*C^T was missing in the original notes.

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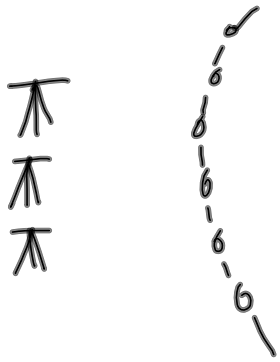

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