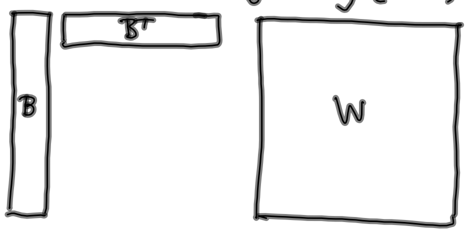
Lecture 40

Sequential formation of NE

40-1

adv: never have to form full B
never have to form full W
for linear models: solution exact
works well if obs. are generated one at a time
or one group at a time
small claim on working memory (RAM)



$\boxed{N}$

for nonlinear models, solution can be exact
if you properly iterate
can add or subtract contributions to NE
can get provisional solution

Dec 2-4:23 PM

disadvantages: not usable if W full

40-2

not exact if NL & don't iterate properly

1. Sequential formation of NE N, t
2. Sequential LS

prior solution

$$Nx = t, \quad x = N^{-1}t, \quad \Sigma_x = Q_x = N^{-1} \quad (\sigma_0^2 = 1)$$

$$N = Q_x^{-1}$$

new obs

$$Bx \approx f \quad (v + Bx = f), \quad W, Q$$

update NE with new data

$$x_n = (N + B^TWB)^{-1}(t + B^TWf)$$

subs expr. from above

$$\underset{new}{\underline{x_n}} = (Q_x^{-1} + B^TWB)^{-1}(Q_x^{-1}\underset{prior}{\underline{x}} + B^TWf)$$

cov. matrix of new param.

$$Q_{x_n} = (Q_x^{-1} + B^TWB)^{-1}, \quad \underline{Q_{x_n}^{-1} = Q_x^{-1} + B^TWB}$$

Dec 2-4:23 PM

Sherman, Morrison, Woodbury, Schur 40-3

matrix inversion lemma

$$(A+UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1}+VA^{-1}U)^{-1}VA^{-1} \leftarrow$$

$$Q_x^{-1}: A,\quad B^T: U,\quad W: C,\quad B: V$$

$$x_n = [Q_x - \underbrace{Q_x B^T (Q+BQ_xB^T)^{-1}}_{K} B Q_x](Q_x^{-1}x + B^T W f)$$

$$x_n = [Q_x - KBQ_x](Q_x^{-1}x + B^T W f)$$

$$x_n = x + Q_x B^T W f - KBx - KBQ_xB^TWf \leftarrow$$

$$Q_{x_n} = Q_x - KBQ_x = \boxed{(I-KB)Q_x = Q_{x_n}}$$

$$K = Q_x B^T (Q + BQ_xB^T)^{-1}$$

$Q_{x_n}Q_{x_n}^{-1}$, WQ insert these into K

Dec 2-4:23 PM

$$K = Q_{x_n}Q_{x_n}^{-1} Q_x B^T W Q \underbrace{(Q+BQ_xB^T)^{-1}}$$ 40-4

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$[(Q+BQ_xB^T)W]^{-1} = (I+BQ_xB^TW)^{-1}$$

$$K = Q_{x_n}(Q_x^{-1}+B^TWB)\, \underbrace{Q_xB^TW}_{\leftarrow} (I+BQ_xB^TW)^{-1}$$

$$K = Q_{x_n}(B^TW + B^TWBQ_xB^TW)(I+BQ_xB^TW)^{-1}$$

$$K = Q_{x_n}B^TW \underbrace{(I+BQ_xB^TW)(I+BQ_xB^TW)^{-1}}$$

$$K = Q_{x_n}B^TW$$

$$x_n = Q_xB^TWf - KBQ_xB^TWf + x - KBx$$

$$x_n = (I-KB)Q_xB^TWf + x - KBx \qquad Q_{x_n} = (I-KB)Q_x$$

Dec 2-4:23 PM

$$X_n = \underbrace{Q_{x_n} B^T W}f + x - KBx \quad 40\text{-}5$$

$$K = Q_{x_n} B^T W$$

$$X_n = Kf + x - KBx$$

$$\boxed{X_n = x + K(f - Bx)}$$

$$\boxed{Q_{x_n} = (I - KB) Q_x}$$

all w/ EMM's notation

Brown Hwang (KF)

$B : H$, $Q_x : P^-$, $Q : R$ $z : f$

$$K = P^- H^T (H P^- H^T + R)^{-1}$$

$$x = x^- + K(z - Hx^-)$$

$$P = (I - KH) P^-$$

$$x^-_{k+1} = \phi \cdot x_k$$

$$P^-_{k+1} = Q_k + \phi P_k \phi^T$$

Dec 2-4:23 PM