Adj.Geospa.Obs. 2016 Exam2 Solution

1. 7 points obs 2 systems (2D) $n = 7\times4 = \underline{\underline{28}}$, $n_0 = 6+7\times2 = \underline{\underline{20}}$, $r = \underline{\underline{8}}$, $u=6$, $c = r+u = 14$

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = \begin{bmatrix} x - a_0 - a_1X - a_2Y \\ y - b_0 - b_1X - b_2Y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & -a_1 & -a_2 \\ 0 & 1 & -b_1 & -b_2 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ v_X \\ v_Y \end{bmatrix} + \begin{bmatrix} -1 & -X & -Y & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -X & -Y \end{bmatrix} \begin{bmatrix} \Delta a_0 \\ \Delta a_1 \\ \Delta a_2 \\ \Delta b_0 \\ \Delta b_1 \\ \Delta b_2 \end{bmatrix} = -F - A\left[\begin{bmatrix} x \\ y \\ X \\ Y \end{bmatrix} - \begin{bmatrix} x^o \\ y^o \\ X^o \\ Y^o \end{bmatrix}\right]$$

$$\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & -5 \end{bmatrix} \begin{bmatrix} V \end{bmatrix} + \begin{bmatrix} -1 & -1 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -2 \end{bmatrix} \begin{bmatrix} \Delta \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & -1 & -5 \end{bmatrix} \left[\begin{bmatrix} 5 \\ 6 \\ 1 \\ 2 \end{bmatrix} - \begin{bmatrix} x^o \\ y^o \\ X^o \\ Y^o \end{bmatrix}\right]$$

$$A \cdot V \quad + \quad B \cdot \Delta = f$$

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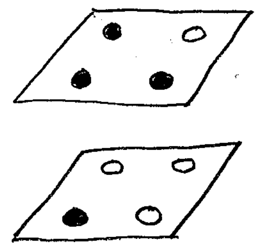
2. total unknowns $= 8\times3 = 24$ (u)
   (a) independent 20 (u')
   (b) dependent 4 (s)
   (c) $s' = s + q$, $q = 4$, $s' = 4+4 = 8$



● = x, y, z independent
O = only x, y independent
z dependent

$4\times3 + 4\times2 = 20$ indep.
$4\times1 = 4$ dep.

$$Z_1 = a_0 + a_1X_1 + a_2Y_1 \qquad Z_5 = a_{00} + a_1X_5 + a_2Y_5$$
$$Z_2 = a_0 + a_1X_2 + a_2Y_2 \qquad Z_6 = a_{00} + a_1X_6 + a_2Y_6$$
$$Z_3 = a_0 + a_1X_3 + a_2Y_3 \qquad Z_7 = a_{00} + a_1X_7 + a_2Y_7$$
$$Z_4 = a_0 + a_1X_4 + a_2Y_4 \qquad Z_8 = a_{00} + a_1X_8 + a_2Y_8$$

added parameters a_0, a_1, a_2, a_{00}
conventional parameters $X_1Y_1Z_1 \cdots X_8Y_8Z_8$

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3. total unknown parameters 6 (u)
   (a) independent 4 (u')
   (b) dependent 2 (s)
   (c) equations:

$$\vec{P}_{12} \cdot \vec{P}_{13} = 0$$

$$\frac{|\vec{P}_{13}|}{|\vec{P}_{12}|} = 3/4$$

$$\begin{bmatrix} X_2 - X_1 \\ Y_2 - Y_1 \end{bmatrix} \cdot \begin{bmatrix} X_3 - X_1 \\ Y_3 - Y_1 \end{bmatrix} = 0$$

$$\frac{\left[(X_3-X_1)^2 + (Y_3-Y_1)^2\right]^{1/2}}{\left[(X_2-X_1)^2 + (Y_2-Y_1)^2\right]^{1/2}} - 3/4 = 0$$

$$(X_2-X_1)(X_3-X_1) + (Y_2-Y_1)(Y_3-Y_1) = 0$$

$$X_2X_3 + X_1^2 - X_2X_1 - X_1X_3 + Y_2Y_3 + Y_1^2 - Y_2Y_1 - Y_1Y_3 = 0$$

$$\frac{\partial F_{c_1}}{\partial X_1} = 2X_1 - X_2 - X_3$$

$$\frac{D_{12}\frac{1}{2}(13)^{-1/2}2(X_3-X_1)(-1) - D_{13}\frac{1}{2}(12)^{-1/2}2(X_2-X_1)(-1)}{D_{12}^2}$$

$$\frac{\partial F_{c_2}}{\partial X_1} = \frac{D_{12}(X_1-X_3)}{D_{13}D_{12}^2} + \frac{D_{13}(X_2-X_1)}{D_{12}D_{12}^2} = \frac{X_1-X_3}{D_{12}D_{13}} + \frac{D_{13}(X_2-X_1)}{D_{12}^3}$$

$$g = \begin{matrix} -\left[(X_2-X_1)(X_3-X_1) + (Y_2-Y_1)(Y_3-Y_1)\right] \\ -\left[\dfrac{\left[(X_3-X_1)^2 + (Y_3-Y_1)^2\right]^{1/2}}{\left[(X_2-X_1)^2 + (Y_2-Y_1)^2\right]^{1/2}} - 3/4\right] \end{matrix}$$

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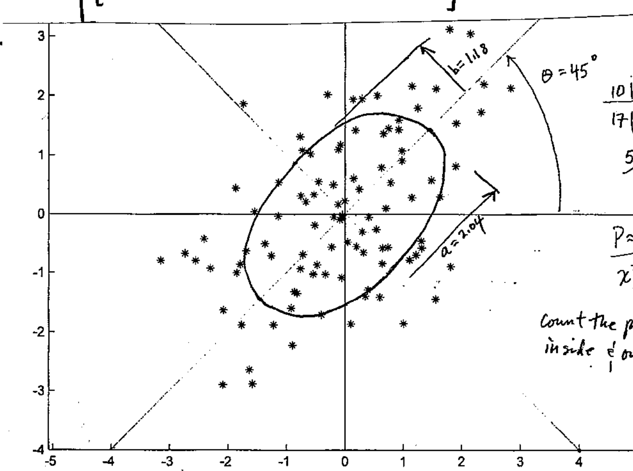
4. $P \approx 50\%$, $\chi^2_{.50,2} = 1.39$

$$a = \sqrt{\lambda_1 \chi^2_{P,2}}, \quad b = \sqrt{\lambda_2 \chi^2_{P,2}}$$

$$\lambda_1 = \frac{a^2}{1.39} = \frac{(2.04)^2}{1.39} = 3$$

$$\lambda_2 = \frac{b^2}{1.39} = \frac{(1.18)^2}{1.39} = 1$$



$\dfrac{10 \mid 14}{17 \mid 10}$

$51/100$

$P \approx 0.50$

$\chi^2_{.50,2} = 1.39$

Count the points inside & outside

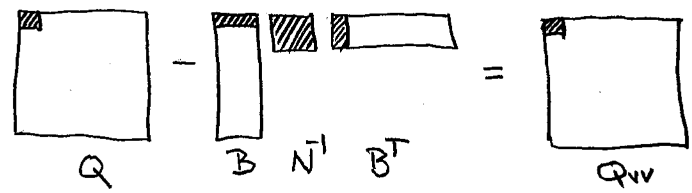
5. $Q_{vv}(\text{ind. obs.}) = Q - BN^{-1}B^T$

$\overline{W} = Q_{vv}W$

$W = I \Rightarrow \overline{W} = Q_{vv}$

redundancy number $r_1 = q_{v_1v_1}$

(partial matrices)



$$N = B^TB = \begin{bmatrix} -1 & -2 & -3 & -4 \\ -1 & -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ -2 & -1 \\ -3 & -1 \\ -4 & -1 \end{bmatrix} = \begin{bmatrix} 30 & 10 \\ 10 & 4 \end{bmatrix}, \quad N^{-1} = \frac{\begin{bmatrix} 4 & -10 \\ -10 & 30 \end{bmatrix}}{(120-100)} = \begin{bmatrix} 0.2 & -0.5 \\ -0.5 & 1.5 \end{bmatrix}$$

$$1 - \begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} .2 & -.5 \\ -.5 & 1.5 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1 - \begin{bmatrix} .3 & -1 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1 - 0.7 = \boxed{0.3}$$

$v_i = r_i \varepsilon_i$, $r_1 = 0.3 \Rightarrow$ 30% of error in obs. 1 will appear in v_1

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6. $$\begin{aligned} v_x - aX - bY - c &= -x \\ v_y + bX - aY - d &= -y \end{aligned} \qquad \begin{bmatrix} v_x \\ v_y \end{bmatrix} + \begin{bmatrix} -X & -Y & -1 & 0 \\ -Y & X & 0 & -1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} -x \\ -y \end{bmatrix}$$

$$b_1 = \begin{bmatrix} -3 & -4 & -1 & 0 \\ -4 & 3 & 0 & -1 \end{bmatrix}, \quad f_1 = \begin{bmatrix} -2 \\ -2.8 \end{bmatrix}$$, sequential formation of N, t

$$N_1 = b_1^Tb_1 = \begin{bmatrix} -3 & -4 \\ -4 & 3 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -3 & -4 & -1 & 0 \\ -4 & 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 25 & 0 & 3 & 4 \\ 0 & 25 & 4 & -3 \\ 3 & 4 & 1 & 0 \\ 4 & -3 & 0 & 1 \end{bmatrix}, \quad t_1 = b_1^Tf_1 = \begin{bmatrix} -3 & -4 \\ -4 & 3 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -2 \\ -2.8 \end{bmatrix} = \begin{bmatrix} 17.2 \\ -0.4 \\ 2 \\ 2.8 \end{bmatrix}$$

$(W = I)$