1. $n = 9$

(a) $\dfrac{n_0 = 4}{r = 5}$

① $\hat{l}_1 - \hat{l}_2 + \hat{l}_6 = 0$

② $\hat{l}_6 + \hat{l}_5 - \hat{l}_7 = 0$

③ $\hat{l}_7 + \hat{l}_4 - \hat{l}_3 = 0$

④ $\hat{l}_2 + \hat{l}_8 - \hat{l}_3 = 0$

⑤ $\hat{l}_1 - \hat{l}_9 - \hat{l}_5 = 0$

obs. only

$V_1 - V_2 + V_6 = -(2.397 - 3.704 + 1.281) = 0.026$ ✓

$V_6 + V_5 - V_7 = -(1.281 + 2.200 - 3.491) = 0.010$ ✓

$V_7 + V_4 - V_3 = -(3.491 + 0.381 - 3.925) = 0.053$ ✓

$V_2 + V_8 - V_3 = -(3.704 + 0.173 - 3.925) = 0.048$ ✓

$V_1 - V_9 - V_5 = -(2.397 - 0.214 - 2.200) = 0.017$ ✓

Keep V_1, V_2, V_3, V_5 make substitutions

Solve for V_4, V_6, V_7, V_8, V_9

$V_1 - V_2 + V_6 = .026$, $\boxed{V_6 = .026 - V_1 + V_2}$

$V_6 + V_5 - V_7 = .010$

$.026 - V_1 + V_2 + V_5 - V_7 = .010$, $-V_1 + V_2 + V_5 - V_7 = -.016$

$\boxed{V_7 = -V_1 + V_2 + V_5 + .016}$

$V_7 + V_4 - V_3 = .053$

$-V_1 + V_2 + V_5 + .016 + V_4 - V_3 = .053$

$V_4 = V_1 - V_2 + V_3 - V_5 + .053 - .016$

$\boxed{V_4 = V_1 - V_2 + V_3 - V_5 + .037}$

$V_4 = V_1 - V_2 - V_5 + V_3 + .037$ ✓✓

$V_6 = -V_1 + V_2 + .026$ ✓✓

$V_7 = -V_1 + V_2 + V_5 + .016$ ✓✓

$V_8 = -V_2 + V_3 + .048$ ✓✓

$V_9 = V_1 - V_5 - .017$ ✓✓

solve ① for V_6
plug into ②, solve for V_7
plug into ③, solve for V_4
Solve ④ for V_8
Solve ⑤ for V_9

$\phi = V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2 + V_6^2 + V_7^2 + V_8^2 + V_9^2$

$\phi = V_1^2 + V_2^2 + V_3^2 + (V_1 - V_2 - V_5 + V_3 + .037)^2 + V_5^2 + (-V_1 + V_2 + .026)^2 + (-V_1 + V_2 + V_5 + .016)^2 +$

$(-V_2 + V_3 + .048)^2 + (V_1 - V_5 - .017)^2$

$\dfrac{\partial \phi}{\partial V_1} = 2V_1 + 2(V_1 - V_2 - V_5 + V_3 + .037) + 2(-V_1 + V_2 + .026)(-1) + 2(-V_1 + V_2 + V_5 + .016)(-1) + 2(V_1 - V_5 - .017) = 0$

$\dfrac{\partial \phi}{\partial V_2} = 2V_2 + 2(V_1 - V_2 - V_5 + V_3 + .037)(-1) + 2(-V_1 + V_2 + .026) + 2(-V_1 + V_2 + V_5 + .016) + 2(-V_2 + V_3 + .048)(-1) = 0$

$\dfrac{\partial \phi}{\partial V_3} = 2V_3 + 2(V_1 - V_2 - V_5 + V_3 + .037) + 2(-V_2 + V_3 + .048) = 0$

$\dfrac{\partial \phi}{\partial V_5} = 2(V_1 - V_2 - V_5 + V_3 + .037)(-1) + 2V_5 + 2(-V_1 + V_2 + V_5 + .016) + 2(V_1 - V_5 - .017)(-1) = 0$

$$5v_1 - 3v_2 + v_3 - 3v_5 = .022 \checkmark$$

$$-3v_1 + 5v_2 - 2v_3 + 2v_5 = .043$$

$$v_1 - 2v_2 + 3v_3 - v_5 = -.085 \checkmark$$

$$-3v_1 + 2v_2 - v_3 + 4v_5 = .004 \checkmark$$

$$
\begin{bmatrix}
5 & -3 & 1 & -3 \\
-3 & 5 & -2 & 2 \\
1 & -2 & 3 & -1 \\
-3 & 2 & -1 & 4
\end{bmatrix}
\begin{bmatrix}
v_1 \\ v_2 \\ v_3 \\ v_5
\end{bmatrix}
=
\begin{bmatrix}
.022 \\ .043 \\ -.085 \\ .004
\end{bmatrix}
,\qquad
\begin{bmatrix}
v_1 \\ v_2 \\ v_3 \\ v_5
\end{bmatrix}
=
\begin{bmatrix}
.014267 \\ .0046 \\ -.029333 \\ .002067
\end{bmatrix}
$$

$$V_4 = V_1 - V_2 - V_5 + V_3 + .037 = .015267$$

$$V_6 = -V_1 + V_2 + .026 = .01633$$

$$V_7 = -V_1 + V_2 + V_5 + .016 = .0084$$

$$V_8 = -V_2 + V_3 + .048 = .014067$$

$$V_9 = V_1 - V_5 - .017 = -.0048$$

$$
V =
\begin{bmatrix}
.014267 \\
.0046 \\
-.02933 \\
.015267 \\
.002067 \\
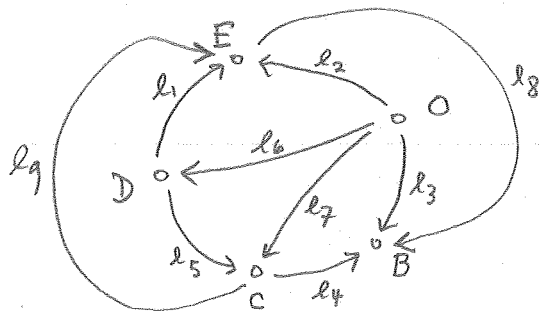.01633 \\
.0084 \\
.014067 \\
-.0048
\end{bmatrix}
$$

$$
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
$$
$$
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1
\end{bmatrix}
$$
$$A$$

if arrow drawn wrong way you can compensate
by negating the value of the observation (which
is difference in elevation)

$$
\hat{\ell} = \ell + V =
\begin{bmatrix}
2.411 \\
3.709 \\
3.896 \\
0.396 \\
2.202 \\
1.297 \\
3.499 \\
0.187 \\
0.209
\end{bmatrix}
$$

1 (b) indirect observations   choose $n_0 = 4$ params B, C, D, E assign $A = 0$

$n = 9$
$n_0 = 4$
$r = 5$

$$l_1 + v_1 = E - D \qquad v_1 = E - D - 2.397$$
$$l_2 + v_2 = E \qquad v_2 = E - 3.704$$
$$l_3 + v_3 = B \qquad v_3 = B - 3.925$$
$$l_4 + v_4 = B - C \qquad v_4 = B - C - 0.381$$
$$l_5 + v_5 = C - D \qquad v_5 = C - D - 2.200$$
$$l_6 + v_6 = D \qquad v_6 = D - 1.281$$
$$l_7 + v_7 = C \qquad v_7 = C - 3.491$$
$$l_8 + v_8 = B - E \qquad v_8 = B - E - 0.173$$
$$l_9 + v_9 = E - C \qquad v_9 = E - C - 0.214$$

$$\begin{bmatrix} .014267 \\ .0046 \\ -.02933 \\ .015267 \\ .002067 \\ .01633 \\ .0084 \\ .014067 \\ -.0048 \end{bmatrix} \checkmark$$

$v^T v = $
$.0018807$

$$\phi = \Sigma v^2 = (E - D - 2.397)^2 + (E - 3.704)^2 + (B - 3.925)^2 + (B - C - .381)^2 + (C - D - 2.200)^2 +$$

$$(D - 1.281)^2 + (C - 3.491)^2 + (B - E - .173)^2 + (E - C - .214)^2$$

$$\frac{\partial \phi}{\partial B} = 2(B - 3.925) + 2(B - C - .381) + 2(B - E - .173) = 0$$

$$\frac{\partial \phi}{\partial C} = 2(B - C - .381)(-1) + 2(C - D - 2.200) + 2(C - 3.491) + 2(E - C - .214)(-1) = 0$$

$$\frac{\partial \phi}{\partial D} = 2(E - D - 2.397)(-1) + 2(C - D - 2.200)(-1) + 2(D - 1.281) = 0$$

$$\frac{\partial \phi}{\partial E} = 2(E - D - 2.397) + 2(E - 3.704) + 2(B - E - .173)(-1) + 2(E - C - .214) = 0$$

$$3B - C + 0D - E = 4.479$$

$$-B + 4C - D - E = 5.096$$

$$0B - C + 3D - E = -3.316$$

$$-B - C - D + 4E = 6.142$$

$$\begin{bmatrix} 3 & -1 & 0 & -1 \\ -1 & 4 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix} \begin{bmatrix} B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 4.479 \\ 5.096 \\ -3.316 \\ 6.142 \end{bmatrix} \qquad \begin{bmatrix} B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 3.89567 \\ 3.4994 \\ 1.29733 \\ 3.7086 \end{bmatrix} \quad v's$$

2.

$$V_1 = a_0 + a_1 + a_2 + a_3 - 1.00 \qquad \phi = \Sigma v^2$$

$$V_2 = a_0 + 2a_1 + 4a_2 + 8a_3 - 1.20$$

$$V_3 = a_0 + 3a_1 + 9a_2 + 27a_3 - 0.75$$

$$V_4 = a_0 + 4a_1 + 16a_2 + 64a_3 + .50$$

$$V_5 = a_0 + 5a_1 + 25a_2 + 125a_3 + 1.00$$

$$V_6 = a_0 + 6a_1 + 36a_2 + 216a_3 + .50$$

$$V_7 = a_0 + 7a_1 + 49a_2 + 343a_3 - 1.00$$

$$\left(a_0 + a_1 + a_2 + a_3 - 1.00\right)^2 + \left(a_0 + 2a_1 + 4a_2 + 8a_3 - 1.20\right)^2 + \left(a_0 + 3a_1 + 9a_2 + 27a_3 - .75\right)^2 +$$

$$\left(a_0 + 4a_1 + 16a_2 + 64a_3 + .50\right)^2 + \left(a_0 + 5a_1 + 25a_2 + 125a_3 + 1.00\right)^2 + \left(a_0 + 6a_1 + 36a_2 + 216a_3 + .50\right)^2 +$$

$$\left(a_0 + 7a_1 + 49a_2 + 343a_3 - 1.00\right)^2 = \Sigma v^2 = \phi$$

$$\frac{\partial \phi}{\partial a_0} = 2\left(a_0 + a_1 + a_2 + a_3 - 1.00\right) + 2\left(a_0 + 2a_1 + 4a_2 + 8a_3 - 1.20\right) + 2\left(a_0 + 3a_1 + 9a_2 + 27a_3 - .75\right)$$
$$2\left(a_0 + 4a_1 + 16a_2 + 64a_3 + .50\right) + 2\left(a_0 + 5a_1 + 25a_2 + 125a_3 + 1.00\right) + 2\left(a_0 + 6a_1 + 36a_2 + 216a_3 + .50\right) +$$
$$2\left(a_0 + 7a_1 + 49a_2 + 343a_3 - 1.00\right) = 0$$

$$\frac{\partial \phi}{\partial a_1} = 2\left(a_0 + a_1 + a_2 + a_3 - 1.00\right) + 2\left(a_0 + 2a_1 + 4a_2 + 8a_3 - 1.20\right)(2) + 2\left(a_0 + 3a_1 + 9a_2 + 27a_3 - .75\right)(3) +$$
$$2\left(a_0 + 4a_1 + 16a_2 + 64a_3 + .50\right)(4) + 2\left(a_0 + 5a_1 + 25a_2 + 125a_3 + 1.00\right)(5) + 2\left(a_0 + 6a_1 + 36a_2 + 216a_3 + .50\right)(6) +$$
$$2\left(a_0 + 7a_1 + 49a_2 + 343a_3 - 1.00\right)(7) = 0$$

$$\frac{\partial \phi}{\partial a_2} = 2\left(a_0 + a_1 + a_2 + a_3 - 1.00\right)^2 + 2\left(a_0 + 2a_1 + 4a_2 + 8a_3 - 1.20\right)(4) + 2\left(a_0 + 3a_1 + 9a_2 + 27a_3 - 0.75\right)(9)$$
$$+ 2\left(a_0 + 4a_1 + 16a_2 + 64a_3 + 0.50\right)(16) + 2\left(a_0 + 5a_1 + 25a_2 + 125a_3 + 1.00\right)(25) + 2\left(a_0 + 6a_1 + 36a_2 + 216a_3 + .50\right)(36)$$
$$+ 2\left(a_0 + 7a_1 + 49a_2 + 343a_3 - 1.00\right)(49) = 0$$

$$\frac{\partial \phi}{\partial a_3} = 2\left(a_0 + a_1 + a_2 + a_3 - 1.00\right) + 2\left(a_0 + 2a_1 + 4a_2 + 8a_3 - 1.20\right)(8) + 2\left(a_0 + 3a_1 + 9a_2 + 27a_3 - .75\right)(27) +$$
$$2\left(a_0 + 4a_1 + 16a_2 + 64a_3 + .50\right)(64) + 2\left(a_0 + 5a_1 + 25a_2 + 125a_3 + 1.00\right)(125) + 2\left(a_0 + 6a_1 + 36a_2 + 216a_3 + .50\right)(216) +$$
$$2\left(a_0 + 7a_1 + 49a_2 + 343a_3 - 1.00\right)(343) = 0$$

now collect coefficients ...

▶ $(1+1+1+1+1+1+1)a_0 + (1+2+3+4+5+6+7)a_1 + (1+4+9+16+25+36+49)a_2 +$

$(1+8+27+64+125+216+343)a_3 + (-1.00-1.20-.75+.50+1.00+1.50-1.00) = 0$

▶ $(1+2+3+4+5+6+7)a_0 + (1+4+9+16+25+36+49)a_1 + (1+8+27+64+125+216+343)a_2 +$

$(1+16+81+256+625+1296+2401)a_3 + (-1.00-2.40-2.25+2.00+5.00+9.00-7.00) = 0$

▶ $(1+4+9+16+25+36+49)a_0 + (1+8+27+64+125+216+343)a_1 + (1+16+81+256+625+1296+2401)a_2 +$

$(1+32+243+1024+3125+7776+16807)a_3 + (-1.00-4.80-6.75+8.00+25.0+18.0-49.0) = 0$

▶ $(1+8+27+64+125+216+343)a_0 + (1+16+81+256+625+1296+2401)a_1 + (1+32+243+1024+3125+7776+16807)a_2 +$

$(1+64+729+4096+15625+46656+117649)a_3 + (-1.00-9.60-20.25+32+125+108-343) = 0$

$7a_0 + 28a_1 + 140a_2 + 784a_3 = 1.95$

$28a_0 + 140a_1 + 784a_2 + 4676a_3 = 2.65$

$140a_0 + 784a_1 + 4676a_2 + 29008a_3 = 10.55$

$784a_0 + 4676a_1 + 29008a_2 + 184820a_3 = 108.85$

$$\Rightarrow \begin{bmatrix} 7 & 28 & 140 & 784 \\ 28 & 140 & 784 & 4676 \\ 140 & 784 & 4676 & 29008 \\ 784 & 4676 & 29008 & 184820 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1.95 \\ 2.65 \\ 10.55 \\ 108.85 \end{bmatrix}$$

$$\overset{B}{\begin{bmatrix} -1 & -1 & -1 & -1 \\ -1 & -2 & -4 & -8 \\ -1 & -3 & -9 & -27 \\ -1 & -4 & -16 & -64 \\ -1 & -5 & -25 & -125 \\ -1 & -6 & -36 & -216 \\ -1 & -7 & -49 & -343 \end{bmatrix}}, \quad \overset{S}{\begin{bmatrix} -1.00 \\ -1.20 \\ -0.75 \\ +0.50 \\ +1.00 \\ +0.50 \\ -1.00 \end{bmatrix}}$$

matrix solution agrees

$a_0 = -0.6143$

$a_1 = 2.5310$

$a_2 = -0.9982$

$a_3 = 0.0958$

substitute a_0, a_1, a_2, a_3 into equations for V_i

$V_1 = .0143$

$V_2 = .0214$

$V_3 = -.1679$

$V_4 = .1714$

$V_5 = .0643$

$V_6 = -.1643$

$V_7 = .0607$

3 (a) $n = 11$ obs. only: designate l_1, l_2, l_3, l_4, l_5 as "basic"

$n_0 = 5$

$r = 6$

express others in terms of these (to avoid having to make substitutions)

$c = r = 6$

$$\hat{l}_7 = \hat{l}_1 + \hat{l}_2$$

$$\hat{l}_8 = \hat{l}_3 + \hat{l}_4 + \hat{l}_5$$

$$\hat{l}_9 = \hat{l}_3 + \hat{l}_4$$

$$\hat{l}_{10} = 360 - \hat{l}_3 - \hat{l}_4 - \hat{l}_5$$

$$\hat{l}_{11} = 360 - \hat{l}_2 - \hat{l}_3 - \hat{l}_4 - \hat{l}_5$$

$$\hat{l}_6 = 360 - \hat{l}_1 - \hat{l}_2 - \hat{l}_3 - \hat{l}_4 - \hat{l}_5$$

$$V_7 = V_1 + V_2 + l_1 + l_2 - l_7$$

$$V_8 = V_3 + V_4 + V_5 + l_3 + l_4 + l_5 - l_8$$

$$V_9 = V_3 + V_4 + l_3 + l_4 - l_9$$

$$V_{10} = -V_3 - V_4 - V_5 + 360 - l_3 - l_4 - l_5 - l_{10}$$

$$V_{11} = -V_2 - V_3 - V_4 - V_5 + 360 - l_2 - l_3 - l_4 - l_5 - l_{11}$$

$$V_6 = -V_1 - V_2 - V_3 - V_4 - V_5 + 360 - l_1 - l_2 - l_3 - l_4 - l_5 - l_6$$

$$V_7 = V_1 + V_2 - 0.33$$

$$V_8 = V_3 + V_4 + V_5 - 0.11$$

$$V_9 = V_3 + V_4 + 0.13$$

$$V_{10} = -V_3 - V_4 - V_5 - 0.12$$

$$V_{11} = -V_2 - V_3 - V_4 - V_5 - 1.15$$

$$V_6 = -V_1 - V_2 - V_3 - V_4 - V_5 + .19$$

$$\phi = \Sigma v^2 = V_1^2 + V_2^2 + V_3^2 + V_4^2 + V_5^2 +$$

$$(-V_1 - V_2 - V_3 - V_4 - V_5 + .19)^2 + (V_1 + V_2 - .33)^2$$

$$(V_3 + V_4 + V_5 - .11)^2 + (V_3 + V_4 + .13)^2 +$$

$$(-V_3 - V_4 - V_5 - .12)^2 + (-V_2 - V_3 - V_4 - V_5 - 1.15)^2$$

all $w_i = 1$,

$$\partial\phi / \partial v_1 = \tfrac{}{}V_1 + \tfrac{}{}(-V_1 - V_2 - V_3 - V_4 - V_5 + .19)(-1) + \tfrac{}{}(V_1 + V_2 - .33) = 0$$

$$\frac{\partial\phi}{\partial V_2} = \tfrac{}{}V_2 + \tfrac{}{}(-V_1 - V_2 - V_3 - V_4 - V_5 + .19)(-1) + \tfrac{}{}(V_1 + V_2 - .33) + \tfrac{}{}(-V_2 - V_3 - V_4 - V_5 - 1.15)(-1) = 0$$

$$\partial\phi / \partial V_3 = \tfrac{}{}V_3 + \tfrac{}{}(-V_1 - V_2 - V_3 - V_4 - V_5 + .19)(-1) + \tfrac{}{}(V_3 + V_4 + V_5 - .11) + \tfrac{}{}(V_3 + V_4 + .13) +$$

$$\tfrac{}{}(-V_3 - V_4 - V_5 - .12)(-1) + \tfrac{}{}(-V_2 - V_3 - V_4 - V_5 - 1.15)(-1) = 0$$

$$\frac{\partial\phi}{\partial V_4} = \tfrac{}{}V_4 + \tfrac{}{}(-V_1 - V_2 - V_3 - V_4 - V_5 + .19)(-1) + \tfrac{}{}(V_3 + V_4 + V_5 - .11) + \tfrac{}{}(V_3 + V_4 + .13) +$$

$$\tfrac{}{}(-V_3 - V_4 - V_5 - .12)(-1) + \tfrac{}{}(-V_2 - V_3 - V_4 - V_5 - 1.15)(-1) = 0$$

$$\frac{\partial\phi}{\partial V_5} = \tfrac{}{}V_5 + \tfrac{}{}(-V_1 - V_2 - V_3 - V_4 - V_5 + .19)(-1) + \tfrac{}{}(V_3 + V_4 + V_5 - .11) + \tfrac{}{}(-V_3 - V_4 - V_5 - .12)(-1) +$$

$$\tfrac{}{}(-V_2 - V_3 - V_4 - V_5 - 1.15)(-1) = 0$$

$(1+1+1)V_1 + (1+1)V_2 + (1)V_3 + (1)V_4 + (1)V_5 = .52$

$(1+1)V_1 + (1+1+1)V_2 + (1+1)V_3 + (1+1)V_4 + (1+1)V_5 = -.63$

$(1)V_1 + (1+1)V_2 + (1+1+1+1+1)V_3 + (1+1+1+1)V_4 + (1+1+1+1)V_5 = -1.10$

$(1)V_1 + (1+1)V_2 + (1+1+1+1)V_3 + (1+1+1+1+1)V_4 + 4V_5 = -1.10$

$(1)V_1 + 2V_2 + 4V_3 + 4V_4 + 5V_5 = -.97$

$3V_1 + 2V_2 + V_3 + V_4 + V_5 = .52$

$2V_1 + 4V_2 + 2V_3 + 2V_4 + 2V_5 = -.63$

$V_1 + 2V_2 + 6V_3 + 5V_4 + 4V_5 = -1.10$

$V_1 + 2V_2 + 5V_3 + 6V_4 + 4V_5 = -1.10$

$V_1 + 2V_2 + 4V_3 + 4V_4 + 5V_5 = -.97$

sym. ✓

$$\begin{bmatrix} 3 & 2 & 1 & 1 & 1 \\ 2 & 4 & 2 & 2 & 2 \\ 1 & 2 & 6 & 5 & 4 \\ 1 & 2 & 5 & 6 & 4 \\ 1 & 2 & 4 & 4 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix} = \begin{bmatrix} .52 \\ -.63 \\ -1.10 \\ -1.10 \\ -.97 \end{bmatrix}$$

$V_1 = .4175$

$V_2 = -.2681$

$V_3 = -.0653$

$V_4 = -.0653$

$V_5 = -.0658$

substitute into earlier expressions for $V_6, V_7, V_8, V_9, V_{10}, V_{11}$

$V_6 = .2369$

$V_7 = -.1806$

$V_8 = -.3064$

$V_9 = -.0006$

$V_{10} = .0764$

$V_{11} = -.6856$

$$V = \begin{bmatrix} .4175 \\ -.2681 \\ -.0653 \\ -.0653 \\ -.0658 \\ .2369 \\ -.1806 \\ -.3064 \\ -.0006 \\ .0764 \\ -.6856 \end{bmatrix} \qquad \hat{\ell} = \begin{bmatrix} 14.587 \\ 25.302 \\ 45.345 \\ 99.665 \\ 70.284 \\ 104.817 \\ 39.889 \\ 215.294 \\ 145.009 \\ 144.706 \\ 119.404 \end{bmatrix}$$

3 (b) need $u = n_0 = 5$ parameters choose x_1, x_2, x_3, x_4, x_5 corresponding to $\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4, \hat{\ell}_5$

$$\ell_1 + V_1 = X_1 \qquad\qquad V_1 = X_1 - 14.17$$
$$\ell_2 + V_2 = X_2 \qquad\qquad V_2 = X_2 - 25.57$$
$$\ell_3 + V_3 = X_3 \qquad\qquad V_3 = X_3 - 45.41$$
$$\ell_4 + V_4 = X_4 \qquad\qquad V_4 = X_4 - 99.73$$
$$\ell_5 + V_5 = X_5 \qquad\qquad V_5 = X_5 - 70.35$$
$$\ell_6 + V_6 = 360 - X_1 - X_2 - X_3 - X_4 - X_5 \qquad V_6 = -X_1 - X_2 - X_3 - X_4 - X_5 + 255.42$$
$$\ell_7 + V_7 = X_1 + X_2 \qquad\qquad V_7 = X_1 + X_2 - 40.07$$
$$\ell_8 + V_8 = X_3 + X_4 + X_5 \qquad\qquad V_8 = X_3 + X_4 + X_5 - 215.60$$
$$\ell_9 + V_9 = X_3 + X_4 \qquad\qquad V_9 = X_3 + X_4 - 145.01$$
$$\ell_{10} + V_{10} = 360 - X_3 - X_4 - X_5 \qquad V_{10} = -X_3 - X_4 - X_5 + 215.37$$
$$\ell_{11} + V_{11} = 360 - X_2 - X_3 - X_4 - X_5 \qquad V_{11} = -X_2 - X_3 - X_4 - X_5 + 239.91$$

$$V = \begin{bmatrix} .4175 \\ -.2708 \\ -.0042 \\ -.1042 \\ -.0825 \\ .2342 \\ -.1833 \\ -.3008 \\ .0217 \\ .0708 \\ -.6883 \end{bmatrix}$$

$$\sigma_{1,2,4,5,6,7,8,9,10,11} = 0.5, \qquad \sigma_3 = 0.1 \qquad \text{let } \sigma_0 = \sigma_1 = 0.5$$

$$W_1 = \frac{\sigma_0^2}{\sigma_1^2} = \frac{(0.5)^2}{(0.5)^2} = 1, \qquad W_3 = \frac{(0.5)^2}{(0.1)^2} = \frac{0.25}{0.01} = 25$$

$$\phi = V_1^2 + V_2^2 + \underline{\underline{25}} V_3^2 + V_4^2 + V_5^2 + V_6^2 + V_7^2 + V_8^2 + V_9^2 + V_{10}^2 + V_{11}^2$$

$$\phi = (X_1 - 14.17)^2 + (X_2 - 25.57)^2 + 25(X_3 - 45.41)^2 + (X_4 - 99.73)^2 + (X_5 - 70.35)^2 +$$
$$(-X_1 - X_2 - X_3 - X_4 - X_5 + 255.42)^2 + (X_1 + X_2 - 40.07)^2 + (X_3 + X_4 + X_5 - 215.60)^2 +$$
$$(X_3 + X_4 - 145.01)^2 + (-X_3 - X_4 - X_5 + 215.37)^2 + (-X_2 - X_3 - X_4 - X_5 + 239.91)^2$$

$$\frac{\partial \phi}{\partial X_1} = 2(X_1 - 14.17) + 2(-X_1 - X_2 - X_3 - X_4 - X_5 + 255.42)(-1) + 2(X_1 + X_2 - 40.07) = 0$$

$$\frac{\partial \phi}{\partial X_2} = 2(X_2 - 25.57) + 2(-X_1 - X_2 - X_3 - X_4 - X_5 + 255.42)(-1) + 2(X_1 + X_2 - 40.07) + 2(-X_2 - X_3 - X_4 - X_5 + 239.91)(-1) = 0$$

$$\frac{\partial \phi}{\partial X_3} = 2 \cdot 25(X_3 - 45.41) + 2(-X_1 - X_2 - X_3 - X_4 - X_5 + 255.42)(-1) + 2(X_3 + X_4 + X_5 - 215.60) + 2(X_3 + X_4 - 145.01) +$$
$$2(-X_3 - X_4 - X_5 + 215.37)(-1) + 2(-X_2 - X_3 - X_4 - X_5 + 239.91)(-1) = 0$$

$$\frac{\partial \phi}{\partial X_4} = 2(X_4 - 99.73) + 2(-X_1 - X_2 - X_3 - X_4 - X_5 + 255.42)(-1) + 2(X_3 + X_4 + X_5 - 215.60) + 2(X_3 + X_4 - 145.01) +$$
$$2(-X_3 - X_4 - X_5 + 215.37)(-1) + 2(-X_2 - X_3 - X_4 - X_5 + 239.91)(-1) = 0$$

$$\frac{\partial \phi}{\partial x_5} = \frac{2}{7}(x_5 - 70.35) + \frac{2}{7}(-x_1 - x_2 - x_3 - x_4 - x_5 + 255.42)(-1) + \frac{2}{7}(x_3 + x_4 + x_5 - 215.60) +$$

$$\frac{2}{7}(-x_3 - x_4 - x_5 + 215.37)(-1) + \frac{2}{7}(-x_2 - x_3 - x_4 - x_5 + 239.91)(-1) = 0$$

$$3x_1 + 2x_2 + x_3 + x_4 + x_5 = 14.17 + 255.42 + 40.07 = 309.66$$

$$2x_1 + 4x_2 + 2x_3 + 2x_4 + 2x_5 = 25.57 + 255.42 + 40.07 + 239.91 = 560.97$$

$$x_1 + 2x_2 + 30x_3 + 5x_4 + 4x_5 = (25)45.41 + 255.42 + 215.60 + 145.01$$
$$+ 215.37 + 239.91 = 2206.56$$

$$x_1 + 2x_2 + 5x_3 + 6x_4 + 4x_5 = 99.73 + 255.42 + 215.60 + 145.01$$
$$+ 215.37 + 239.91 = 1171.04$$

$$x_1 + 2x_2 + 4x_3 + 4x_4 + 5x_5 = 70.35 + 255.42 + 215.60 + 215.37$$
$$+ 239.91 = 996.65$$

$$\begin{bmatrix} 3 & 2 & 1 & 1 & 1 \\ 2 & 4 & 2 & 2 & 2 \\ 1 & 2 & 30 & 5 & 4 \\ 1 & 2 & 5 & 6 & 4 \\ 1 & 2 & 4 & 4 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 309.66 \\ 560.97 \\ 2206.56 \\ 1171.04 \\ 996.65 \end{bmatrix} \quad , \quad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 14.5875 \\ 25.29917 \\ 45.4058 \\ 99.62583 \\ 70.2675 \end{bmatrix}$$

(see V on previous page)

$$\hat{\ell} = \begin{bmatrix} 14.587 \\ 25.299 \\ 45.406 \\ 99.626 \\ 70.268 \\ 104.814 \\ 39.887 \\ 215.299 \\ 145.032 \\ 144.701 \\ 119.402 \end{bmatrix}$$