Adv. Geospa. Est. Exam 1 (Do Over)

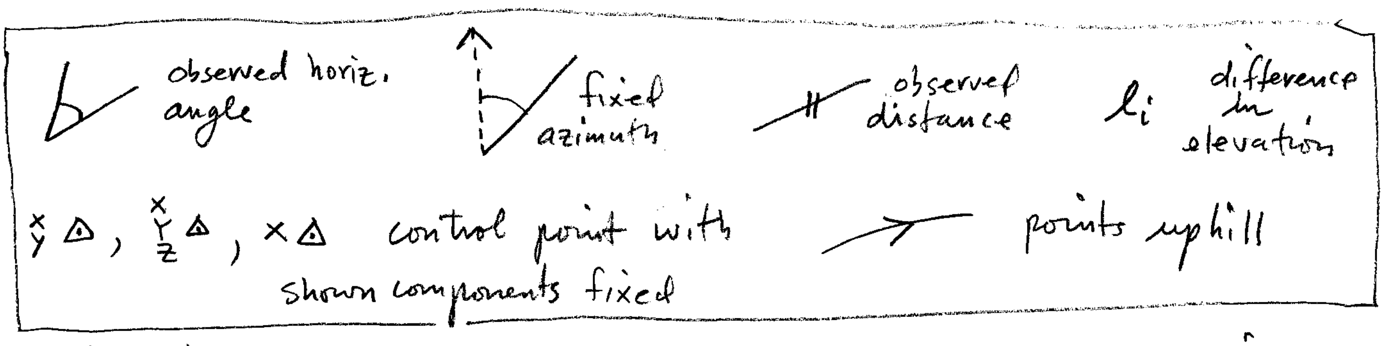
27 April 2009

(1 sheet of notes allowed)

Name ____________________

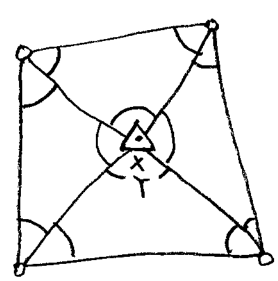
1. You observed a single quantity multiple times and obtained a least squares estimate: $X = 5.0$, $\sigma_X = 0.1$. You make 2 new observations $l_1 = 4.8$, $l_2 = 4.9$ both with $\sigma_l = 0.2$. (a) Using indirect observations and unified L.S. find the new estimate of X. (b) Is there an easier way to solve this problem? If yes, show it.

2.

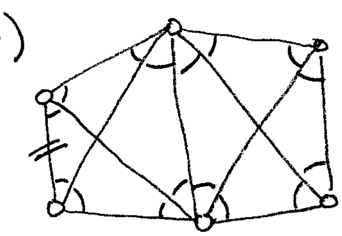


Using above symbol meanings, what are the number of minimal constraints needed for the following networks?

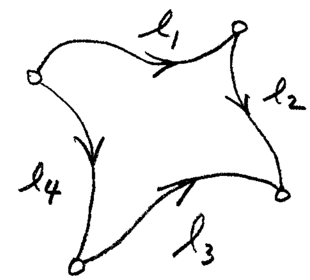
(a) 2D



(b) 2D

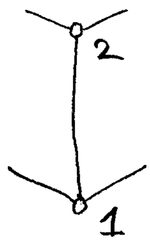


(c) 1D



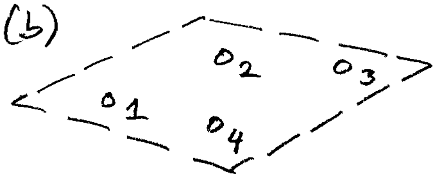
(note: minimal constraints = tie to reference coordinate system)

3. (a)



Write constraint equation(s) to force points 1 & 2 to lie on a vertical line. Show in matrix form (without added parameters).

(b)



Write constraint equation(s) to force points 1 → 4 to line in a plane. Show in matrix form (with added parameters).

4. The following steps occur in the development of constraint solution by elimination:

$$Av + B\Delta = f$$
$$C\Delta = g$$

for elimination of constraints we partition Δ into Δ_1, Δ_2

$$Av + B_1\Delta_1 + B_2\Delta_2 = f$$
$$C_1\Delta_1 = C_2\Delta_2 = g$$

Solve symbolically for Δ_1

$$\Delta_1 = C_1^{-1}(g - C_2\Delta_2)$$

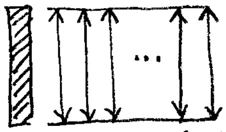
subsequent steps allow us to solve for Δ_2 and Δ_1.

if $Q_{\Delta_2\Delta_2}$ is determined, what is an expression for $Q_{\Delta_1\Delta_1}$?

Adv. Geospa. Est. Exam 1 (b)

SOLUTION — 29 April 2009

1. multiple obs of single quantity, w/ prior estimate & cov.



$\ell \quad \ell_1\,\ell_2$

prior new

prior $x = 5$, $\sigma_x = 0.1$

new $\ell_1 = 4.8$, $\ell_2 = 4.9$, $\sigma = 0.2$

I/O: $\ell + v = x$, $v - x = -\ell$, $\sigma_0^2 = 1$ (assume)

(a)

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \end{bmatrix}[x] = \begin{bmatrix} -\ell_1 \\ -\ell_2 \end{bmatrix}$$

$$v + B\Delta = f$$

$$W = \begin{bmatrix} \frac{1}{(0.2)^2} & 0 \\ 0 & \frac{1}{(0.2)^2} \end{bmatrix} = \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}, \quad W_{xx} = \frac{1}{(0.1)^2} = 100$$

$$\boxed{\Delta = (N + W_{xx})^{-1}(t + W_{xx}x)}$$

linear ULS

$$N = B^T W B = \begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix} = 50$$

$$t = B^T W f = \begin{bmatrix} -1 & -1 \end{bmatrix}\begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix}\begin{bmatrix} -4.8 \\ -4.9 \end{bmatrix} = 242.5$$

$$\Delta = (50 + 100)^{-1}(242.5 + 100 \cdot 5) = 742.5/150 = \underline{\underline{4.95}}$$

(b) other ways; just LS I/O no ULS:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}(x) = \begin{bmatrix} -\ell_1 \\ -\ell_2 \\ -\ell_3 \end{bmatrix}, \quad f = \begin{pmatrix} -4.8 \\ -4.9 \\ -5.0 \end{pmatrix}, \quad W = \begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

$$v + B\Delta = f$$

$$N = B^T W B = \begin{bmatrix} -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 25 & & \\ & 25 & \\ & & 100 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} = 150$$

$$t = B^T W f = \begin{bmatrix} -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 25 & & \\ & 25 & \\ & & 100 \end{bmatrix}\begin{bmatrix} -4.8 \\ -4.9 \\ -5.0 \end{bmatrix} = 742.5$$

$$\Delta = N^{-1}t = \frac{742.5}{150} = \underline{\underline{4.95}}$$

or, just weighted mean: $\hat{x} = \dfrac{w_1 x_1 + w_2 x_2 + w_3 x_3}{w_1 + w_2 + w_3}$

$$\hat{x} = \frac{25 \cdot 4.8 + 25 \cdot 4.9 + 100 \cdot 5.0}{25 + 25 + 100} = 742.5/150 = \underline{\underline{4.95}}$$

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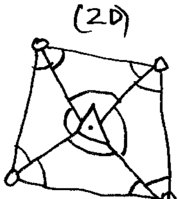
2. (a) (2D)

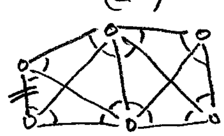


$\#mc = 2$

have T_x, T_y

need Scale & Rot.

(b) (2D)



$\#mc = 3$

have scale

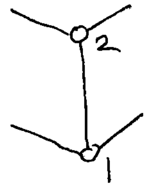
need T_x, T_y, Rot.

(c) (1D)

$\#mc = 1$

have scale

need T_z or absolute height

3. (a) 

intended as 3D, but can interpret as 2D or 3D

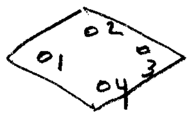
(no added params.)

2D: $X_2 - X_1 = 0$

$$\begin{bmatrix} -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{bmatrix} = 0$$

3D: $X_2 - X_1 = 0$
$Y_2 - Y_1 = 0$

$$\begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(b) 

(w/ added parameters.)

$Z = a_0 + a_1 X + a_2 Y$

$F_i = z_i - a_0 - a_1 x_i - a_2 y_i$

$x_i y_i z_i$: parameters

a_0, a_1, a_2 : added parameters

$$\begin{bmatrix} -a_1 & -a_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -x_1 & -y_1 \\ 0 & 0 & 0 & -a_1 & -a_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -x_2 & -y_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_1 & -a_2 & 1 & 0 & 0 & 0 & -1 & -x_3 & -y_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_1 & -a_2 & 1 & -1 & -x_4 & -y_4 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \\ \vdots \\ \Delta x_4 \\ \Delta y_4 \\ \Delta z_4 \\ \Delta a_0 \\ \Delta a_1 \\ \Delta a_2 \end{bmatrix} = \begin{bmatrix} -F_1^0 \\ -F_2^0 \\ -F_3^0 \\ -F_4^0 \end{bmatrix}$$

$$C \Delta = g$$

OR,

$$\underbrace{\begin{bmatrix} -a_1 & -a_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a_1 & -a_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -a_1 & -a_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -a_1 & -a_2 & 1 \end{bmatrix}}_{D_1} \cdot \underbrace{\begin{bmatrix} \Delta x_1 \\ \Delta y_1 \\ \Delta z_1 \\ \vdots \\ \Delta x_4 \\ \Delta y_4 \\ \Delta z_4 \end{bmatrix}}_{\Delta} + \underbrace{\begin{bmatrix} -1 & -x_1 & -y_1 \\ -1 & -x_2 & -y_2 \\ -1 & -x_3 & -y_3 \\ -1 & -x_4 & -y_4 \end{bmatrix}}_{D_2} \cdot \underbrace{\begin{bmatrix} \Delta a_0 \\ \Delta a_1 \\ \Delta a_2 \end{bmatrix}}_{\Delta'} = \underbrace{\begin{bmatrix} -F_1^0 \\ -F_2^0 \\ -F_3^0 \\ -F_4^0 \end{bmatrix}}_{h}$$

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4. $Q_{\Delta_1 \Delta_1} = ?$ $\quad Q_{\Delta_2 \Delta_2}$ known

$$\Delta_1 = C_1^{-1}(g - C_2 \Delta_2)$$

$$\Delta_1 = \boxed{-C_1^{-1} C_2} \Delta_2 + C_1^{-1} g$$

($Y = AX + b$ for error prop.)

$$Q_{\Delta_1 \Delta_1} = (-C_1^{-1} C_2) Q_{\Delta_2 \Delta_2} (-C_1^{-1} C_2)^T$$

$$Q_{\Delta_1 \Delta_1} = C_1^{-1} C_2 Q_{\Delta_2 \Delta_2} C_2^T (C_1^{-1})^T$$