1-1

n observations, n_0 minimum

$r = n - n_0$

if no parameters then $c = r$

if you add u independent parameters

$c = r + u$

---

if add u_2 dependent parameters

$\Delta = \begin{bmatrix} \Delta_1 \\ u_1 1 \\ \Delta_2 \\ u_2, 1 \end{bmatrix}$ $(u_2 = s)$

$c = r + u_1$

$c + s = r + \underbrace{u_1 + s}_{u}$ $\quad c + s = r + u$

for each of dependent parameters, need

1 constraint eqn $\Rightarrow s (= u_2)$ constr. eqn.

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1-2

Solve LS problem w/ constraints

1. conventional approach
   - problem solvable w/o constr.
   - problem not solvable w/o constr.
2. elimination
3. added parameters

$Av + B\Delta = f$ (Gen LS)

$C\Delta = g$

$\Phi' = v^T W v - 2k^T (Av + B\Delta - f) - 2k_c^T (C\Delta - g)$

augmented obj. function

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1-3

$\frac{\partial \phi'}{\partial v}$ ......=0, transpose, negate

$\frac{\partial \phi'}{\partial k}$ ...=0, ", "

$\frac{\partial \phi'}{\partial \Delta}$ ...=0, ", "

$\frac{\partial \phi'}{\partial k_c}$ ...=0, ", "

$$\left.\begin{aligned} -Wv + A^T k &= 0 \\ \rightarrow \quad Av + B\Delta &= f \\ B^T k + C^T k_c &= 0 \\ \rightarrow \quad C\Delta &= g \end{aligned}\right\} \begin{bmatrix} -W & A^T & 0 & 0 \\ A & 0 & B & 0 \\ 0 & B^T & 0 & C^T \\ 0 & 0 & C & 0 \end{bmatrix} \begin{bmatrix} v \\ k \\ \Delta \\ k_c \end{bmatrix} = \begin{bmatrix} 0 \\ f \\ 0 \\ g \end{bmatrix}$$

full normal eqn's (sym)

$Wv = A^T k$, $v = QA^T k$

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1-4

$v = QA^T k$

$Av + B\Delta = f \Rightarrow AQA^T k + B\Delta = f$

$Q_e k + B\Delta = f$

$k + W_e(B\Delta) = W_e f$

$k = W_e(f - B\Delta)$

$B^T k + C^T k_c = 0$

$B^T W_e (f - B\Delta) + C^T k_c = 0$

$-\underbrace{B^T W_e B}_{N}\Delta + \underbrace{B^T W_e f}_{t} + C^T k_c = 0$

$-N\Delta + t + C^T k_c = 0$

if N full rank, then solvable w/o constraints

$-N\Delta = -t - C^T k_c$

$N\Delta = t + C^T k_c$

$\Delta = \underbrace{N^{-1} t}_{\Delta^0} + \underbrace{N^{-1} C^T k_c}_{\delta\Delta}$

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if N not full rank : prob not solvable w/o constr. 1-5

$$-N\delta + C^T k_c = -t$$
$$C\delta = g$$
$$\Rightarrow \begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} \delta \\ k_c \end{bmatrix} = \begin{bmatrix} -t \\ g \end{bmatrix}$$

this method always works

---

for fullrank N :

$$\delta = N^{-1}t + N^{-1}C^T k_c$$

$$C\delta = g$$

$$C\left[N^{-1}t + N^{-1}C^T k_c\right] = g$$

$$CN^{-1}t + CN^{-1}C^T k_c = g$$

$$k_c = (CN^{-1}C^T)^{-1}(g - CN^{-1}t)$$ derive this

$$Q_{\delta\delta} = N^{-1}(I - C^T(CN^{-1}C^T)^{-1}CN^{-1})$$

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for non full rank N, 1-6

$$\begin{bmatrix} -N & C^T \\ C & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \alpha & \beta^T \\ \beta & \gamma \end{bmatrix}$$ then

$$Q_{\delta\delta} = -\alpha$$

derived in OLS

Solving by Elimination (of dependent parameters)

$$Av + B\delta = f$$
$$C\delta = g$$

partition $\delta = \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}$ δ_1 is dependent

$$Av + B_1\delta_1 + B_2\delta_2 = f$$
$$C_1\delta_1 + C_2\delta_2 = g$$

OLS $C_{11} = C_1$

partition δ such that C_1 square + full rank
(actually we partition C and δ)

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$$Av + B_1\delta_1 + B_2\delta_2 = f$$
$$C_1\delta_1 + C_2\delta_2 = g$$

1-7

$$* \quad \delta_1 = C_1^{-1}(g - C_2\delta_2)$$

$$Av + B_1C_1^{-1}(g - C_2\delta_2) + B_2\delta_2 = f$$
$$Av + -B_1C_1^{-1}C_2\delta_2 + B_2\delta_2 = f - B_1C_1^{-1}g$$
$$Av + \underbrace{(B_2 - B_1C_1^{-1}C_2)}_{\bar{B}}\delta_2 = \underbrace{f - B_1C_1^{-1}g}_{\bar{f}}$$

$$Av + \bar{B}\delta_2 = \bar{f}$$

solve this as unconstrained LS problem

then

$$\delta_1 = C_1^{-1}(g - C_2\delta_2)$$

and $v = QA^T k$

$$k = W_e(\bar{f} - \bar{B}\delta_2)$$

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