CE 697 Adv. Data Adj. lect. 05 5-1

Next session is next <u>Tuesday</u>

Derive F statistic for constraint check

$\Sigma = \sigma_0^2 Q$ $(C\delta - g)$

$Q = \frac{1}{\sigma_0^2}\Sigma$ constrained LS g: constant

$\Sigma^{-1} = \frac{1}{\sigma_0^2} W$ look @ fit of constrained w/ prior adj.

$W = \sigma_0^2 \Sigma^{-1}$ g: RV

$C\delta = g$, $C\delta - g = 0$ $\frac{v^T W v}{\sigma_0^2} \sim \chi^2_r$

$Q_{gg} = C Q_{\delta\delta} C^T$

$\Sigma_{gg} = \sigma_0^2 C Q_{\delta\delta} C^T$

$\Sigma_{gg}^{-1} = \frac{1}{\sigma_0^2}(C Q_{\delta\delta} C^T)^{-1}$

$(x-\mu_x)^T \Sigma_{xx}^{-1} (x-\mu_x) \sim \chi^2_n$

$g^T \Sigma_{gg}^{-1} g \sim \chi^2_s$

$\frac{1}{\sigma_0^2}(C\delta - g)^T (C N^{-1} C^T)^{-1} (C\delta - g) \sim \chi^2_s$

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$$F_{v_1,v_2} = \frac{\chi^2_{v_1}/v_1}{\chi^2_{v_2}/v_2}$$ 5-2

$$\frac{\dfrac{\cancel{\frac{1}{\sigma_0^2}}(C\delta-g)^T(CN^{-1}C^T)^{-1}(C\delta-g)}{s}}{\dfrac{v^TWv}{\cancel{\sigma_0^2}}\cdot\dfrac{1}{r}} \sim F_{s,r}$$

$$\frac{(C\delta-g)^T(CN^{-1}C^T)^{-1}(C\delta-g)}{s\,\hat{\sigma}_0^2} \sim F_{s,r}$$ if σ_0^2 unknown

if σ_0^2 known

$$\frac{1}{\sigma_0^2}(C\delta-g)^T(CN^{-1}C^T)^{-1}(C\delta-g) \sim \chi^2_s$$

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monte carlo simulation

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$$\frac{V^T W V}{\sigma_0^2} \sim \chi_r^2$$

steps to make m.c. sim.

```
select σ  (=σ0^2)
gen. perfect observations
hs = zeros(50,000 , 1)

for i = 1:50,000
    obs = perf.obs + σ * randn(n,1)
    do LS adj
    hs(i) = VᵀWV/σ0^2
    end
M=30
[H,x] = hist (hs, M)
binw = x(2) - x(1)
area = 0
```

$\underbrace{\sigma * randn(n,1)}_{errors}$

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```
for i = 1:M
    area = area + binw * H(i)
    end
pH = H/area
bar (x, pH)
xx = 0 : 0.1 : 30
yy = pdf('chi2', xx, r)
plot(xx,yy)
```

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Unified LS

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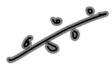
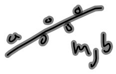
conventional line fit  $n=5, n_0=3$ $r=3$

x = const, y obs.

I/O $u=2, c=5$

% $u=0, c=3$

prior knowledge for parameters (m, b) & σ  $n=7, n_0=2, r=5$

GLS, $u=2$, $c=7$

$$y_1+v_1 = Mx_1+B$$
$$y_2+v_2 = Mx_2+B$$
$$\vdots$$
$$y_5+v_5 = Mx_5+B$$
$$m = M$$
$$b = B$$

---

obs only $r=5$, $u=0$, $c=5$

$$\left.\begin{aligned} y_1+v_1 &= mx_1+b \\ &\vdots \\ y_5+v_5 &= mx_5+b \end{aligned}\right\}$$

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Unified LS derivation (linear)

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$$A(\underset{\substack{\uparrow \\ obs}}{l}+v) + B(\underset{\substack{\uparrow \\ obs}}{x}+\delta) = d$$

$$Av + B\delta = d - Al - Bx = \bar{f}$$

$$\begin{bmatrix} A & B \end{bmatrix}\begin{bmatrix} v \\ \delta \end{bmatrix} = \bar{f}$$

$$\bar{A} \quad \bar{v} = \bar{f}$$

$$\bar{Q} = \begin{bmatrix} Q & 0 \\ 0 & Q_{xx} \end{bmatrix}$$ assume uncorrelated

(recall $k = W_e f$, $v = QA^T k$)

$$\bar{v} = \bar{Q}\bar{A}^T(\bar{A}\bar{Q}\bar{A}^T)^{-1}\bar{f}$$

$$= \bar{Q}\bar{A}^T\left(\begin{bmatrix} A & B \end{bmatrix}\begin{bmatrix} Q & 0 \\ 0 & Q_{xx} \end{bmatrix}\begin{bmatrix} A^T \\ B^T \end{bmatrix}\right)^{-1}\bar{f}$$

$$= \bar{Q}\bar{A}^T(AQA^T + BQ_{xx}B^T)^{-1}f$$

$$\bar{v} = \begin{bmatrix} v \\ \delta \end{bmatrix} = \begin{bmatrix} Q & 0 \\ 0 & Q_{xx} \end{bmatrix}\begin{bmatrix} A^T \\ B^T \end{bmatrix}(Q_e + BQ_{xx}B^T)^{-1}f$$

$$v = QA^T(Q_e + BQ_{xx}B^T)^{-1}f$$

$$\delta = Q_{xx}B^T\underbrace{(Q_e + BQ_{xx}B^T)}^{-1}f$$

Sherman-Morrison matrix inversion lemma

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$$(A+UCV)^{-1} = A^{-1} - A^{-1}U\left(C^{-1}+VA^{-1}U\right)^{-1}VA^{-1} \qquad 5\text{-}7$$

$$\Delta = Q_{xx}B^T\underbrace{\left(Q_e + BQ_{xx}B^T\right)}^{-1}\bar{f}$$

$$\Delta = Q_{xx}B^T\left[Q_e^{-1} - Q_e^{-1}B\left(Q_{xx}^{-1}+B^TQ_e^{-1}B\right)^{-1}B^TQ_e^{-1}\right]\bar{f}$$

$$= Q_{xx}B^TQ_e^{-1}\left[I - B\left(B^TQ_e^{-1}B + Q_{xx}^{-1}\right)^{-1}B^TQ_e^{-1}\right]\bar{f}$$

$$= Q_{xx}B^TW_e\left[I - B\left(B^TW_eB + W_{xx}\right)^{-1}B^TW_e\right]\bar{f}$$

$$= Q_{xx}\underbrace{B^TW_e\bar{f}}_{t} - Q_{xx}\underbrace{B^TW_eB}_{N}\left(\underbrace{B^TW_eB}_{N} + W_{xx}\right)^{-1}\underbrace{B^TW_e\bar{f}}_{t}$$

$$\Delta = Q_{xx}t - Q_{xx}N\left(N+W_{xx}\right)^{-1}t \qquad \cancel{N, t}$$