Lect. 06 Adv. Data Adj. 6-1

HW3 note: $A \cdot B = I$ A_{11} A_{22} A, B

s,s m,m $s+m, s+m$

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} I_s & 0 \\ 0 & I_m \end{bmatrix}$$

in book uses A_{22}

you don't using A_{11} pivot

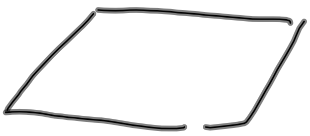
hints for HW2:

plane fit: $ax + by + cz + d = 0$

$z = ax + by + d$ } choose axis most $\perp$

$x = by + cz + d$ } to plane & eliminate

$\vdots$ that coefficient



$z = ax + by + d$

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other HW2 problem: inner constraints 6-2

all M.C. parameter values will be different

all residuals will be same

shape of conf. ellipse is important result

Next meeting will be Tuesday 21st

=

finish ULS derivation from last time

linear LS problem (GLS) l: obs., v: residuals

$A(l+v) + B(x+\delta) = d$ x: obs. of parameters, Q_{xx} or W_{xx}

$Av + B\delta = d - Al - Bx$ δ: residual for x Σ_{xx}

$\hat{x} = x + \Delta$

$[A \; B] \begin{bmatrix} v \\ \delta \end{bmatrix} = \bar{f}$ $\Rightarrow$ looks like obs. only

$\vdots$

$\Delta = Q_{xx}\bar{t} - Q_{xx} N (N + W_{xx})^{-1} \bar{t}$, $\bar{t} = B^T W_e \bar{f}$

apply matrix identity (A70)

$(A+B)^{-1} = A^{-1}(A^{-1} + B^{-1})^{-1} B^{-1}$

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$$\Delta = Q_{xx}\bar{t} - Q_{xx} N N^{-1}(N^{-1}+Q_{xx})^{-1} Q_{xx}\bar{t}$$

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$$= \left[Q_{xx} - Q_{xx}\underline{(N^{-1}+Q_{xx})^{-1}}Q_{xx}\right]\bar{t}$$

apply S.M. inverse formula

this part of derivation is not correct – see the following inserted page for the correct derivation steps

$$\Delta = \left[Q_{xx} - Q_{xx}\left(N - NI(Q_{xx}^{-1}+INI)^{-1}I\cdot N\right)Q_{xx}\right]\bar{t}$$

$$\left[Q_{xx} - Q_{xx}\left(N - N(N+W_{xx})^{-1}N\right)Q_{xx}\right]\bar{t}$$

$$Q_{xx}\bar{t} - \left(I - (N+W_{xx})^{-1}N \; Q_{xx}\right)\bar{t}$$

?? $N^{-1} = Q_{00}$

$$\Delta = \cancel{Q_{xx}\bar{t}} - \cancel{Q_{xx}\bar{t}} + (N+W_{xx})^{-1}\bar{t}$$

$$\boxed{\Delta = (N+W_{xx})^{-1}\bar{t}}$$

$\bar{t} = B^T W_e \bar{f}$

$\bar{f} = d - A\ell - Bx$

if $x=0$, $W_{xx}=0$, reverts to conventional solution

$$\delta = N^{-1}t$$

$$v = QA^T W_e(\bar{f} - B\delta)$$

concludes derivation #1

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new inserted page (following lecture) to correct derivation of ULS (deriv. #1)

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$$\Delta = \left[Q_{xx} - Q_{xx}\underline{(N^{-1}+Q_{xx})^{-1}}Q_{xx}\right]\bar{t}$$

$\Downarrow$

using Sherman-Morrison you can express this in 2 ways, on previous page I chose the wrong way, now choose the other way:

$$\Delta = \left[Q_{xx} - Q_{xx}\underline{\left(W_{xx} - W_{xx}(N+W_{xx})^{-1}W_{xx}\right)}Q_{xx}\right]\bar{t}$$

$$= \left[Q_{xx} - \left(I - (N+W_{xx})^{-1}W_{xx}\right)Q_{xx}\right]\bar{t}$$

$$= \left[Q_{xx} - \left(Q_{xx} - (N+W_{xx})^{-1}\right)\right]\bar{t}$$

$$= \left[\cancel{Q_{xx}} - \cancel{Q_{xx}} + (N+W_{xx})^{-1}\right]\bar{t}$$

$$\boxed{\Delta = (N+W_{xx})^{-1}\bar{t}}$$

; for completeness repeat some of the related equations

$\bar{t} = B^T W_e \bar{f}$

$\bar{f} = d - A\ell - Bx$

$v = QA^T W_e(\bar{f} - B\Delta)$

This concludes derivation #1, linear ULS.

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Derivation #2 $\hat{x} = x + \delta$, $\bar{f} = d - Al - Bx$ 6-5
(linear) $\Delta = v_x$

$$\delta = (N + W_{xx})\bar{t} \;, \quad \bar{t} = B^T W_e \bar{f}$$

$$v = QA^T W_e (\bar{f} - B\delta)$$

Derivation #3 $\hat{x} = x + \delta = x + v_x$

$$\bar{f} = d - Al - Bx$$

$$\dot{f} = \begin{bmatrix} \bar{f} \\ 0 \end{bmatrix}, \quad \dot{t} = \dot{B}^T \dot{W}_e \dot{f} = B^T W_e \bar{f}$$

(same as prior $\bar{t}$)

$$\boxed{\begin{aligned} \delta &= (N + W_{xx})^{-1} \dot{t} \\ v &= QA^T W_e (\bar{f} - B\delta) \end{aligned}}$$

$$\dot{B} = \begin{bmatrix} B \\ -I \end{bmatrix}, \quad \dot{W}_e = \begin{bmatrix} W_e & 0 \\ 0 & W_{xx} \end{bmatrix}$$

$$\Delta = v_x$$

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Deriv. #4 Δ: total parameter 6-6
(linear) $\hat{x} = x + v_x = \Delta$

$$Av + B\delta = f$$

$$v_x - \Delta = -x$$

$$\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} v \\ v_x \end{bmatrix} + \begin{bmatrix} B \\ -I \end{bmatrix} \delta = \begin{bmatrix} f \\ -x \end{bmatrix}$$

$$\dot{A} \;\; \dot{v} \;+\; \dot{B}\Delta = \dot{f} \;, \quad f = d - Al$$

$$\Delta = (N + W_{xx})^{-1} (t + W_{xx} \cdot x)$$

$$v = QA^T W_e (f - B\delta)$$

$$v_x = \Delta - x$$

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Nonlinear ULS $F(\ell, x) = 0$ 6-7

(based on Deriv. #4)

$$F(\ell, x) \approx F(\ell^\circ, x^\circ) + \frac{\partial F}{\partial \ell}\delta\ell + \frac{\partial F}{\partial x}\delta x = 0$$

$$\underbrace{\frac{\partial F}{\partial \ell}}_{A}\delta\ell + \underbrace{\frac{\partial F}{\partial x}}_{B}\delta x = -F(\ell^\circ, x^\circ)$$

$$\ell + v = \ell^\circ + \delta\ell$$

$$\delta\ell = \ell - \ell^\circ + v$$

$$A(\ell - \ell^\circ + v) + B\delta x = -F(\ell^\circ, x^\circ)$$

$$Av + B\Delta = -F(\ell^\circ, x^\circ) - A(\ell - \ell^\circ)$$

$$\boxed{\begin{aligned} Av + B\Delta &= f \\ v_x - \Delta &= f_x \end{aligned}}$$

cond. eqn's

$$\hat{x} = x + v_x = x^\circ + \Delta$$

$$v_x - \Delta = \underbrace{x^\circ - x}_{f_x}$$

$$\begin{bmatrix} A & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} v \\ v_x \end{bmatrix} + \begin{bmatrix} B \\ -I \end{bmatrix}\Delta = \begin{bmatrix} f \\ f_x \end{bmatrix}$$

$$\boxed{\begin{aligned} \Delta &= (N + W_{xx})^{-1}(t - W_{xx} \cdot f_x) \\ v &= QA^T W_e (f - B\Delta) \end{aligned}}$$

$\Delta = N^{-1}t$

similar to classic solution

Counting: ℓ $n,1$ 6-8

x $\mu,1$

$r = n + \mu - n_0$

be cautious if prior Q_x is large number then you are not really adding redundancy

$$Q_{\Delta\Delta} = (N + W_{xx})^{-1}$$

$$\hat{\sigma}_0^2 = \frac{v^T W v + v_x^T W_{xx} v_x}{r}$$