Lecture 09 28 June 2016 9-1
Adv. Data Adj. CE697

next class 1 week 5 July Tuesday
today IRLS & RANSAC
next week L1 norm minimization
Then Seq. LS & KF
IRLS = Iteratively Reweighted Least Squares
not based on statistical theory, but it is
(a) easy to implement
(b) usually working well (provided r is good)
Lin./Non Lin. iterate until convergence *
Then (weighting scheme)

$$w_{k+1} = w_k * \begin{cases} 1 & \text{if } |v_{k,i}| \le 3\sigma_i \\ e^{-|v_{k,i}|/3\sigma_i} & \text{if } |v_{k,i}| > 3\sigma_i \end{cases}$$

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iterate until stable
no change in parameters
no change in $\hat{v}^T W v$
bad observations gradually get de-weighted until they have no influence on parameter estimation
good observations, while maybe initially having large residuals are gradually resurrected until residuals become acceptable

IRLS usually used as a diagnostic tool
remove bad observations and re-run with correction of LS

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alt. wt. #1

$$W_{k+1} = W_k * \begin{cases} 1 & \text{if } |v| \le 2\sigma_i \\ e^{-v^2} & \text{if } |v| > 2\sigma_i \end{cases}$$

alt wt. #2

get converged solution next 2 iterations $W_i = \left\{ e^{-\left[\frac{|v|}{\sigma}\right]^{4.4}} \right\}^{0.05}$

subsequent iterations $W_i = \left\{ e^{-\left[\frac{|v|}{\sigma}\right]^{3.0}} \right\}^{0.05}$

IRLS = Damped Methods

for alt. wt. #2 the "iterations" refer to iterations inside the NL newton process

This technique is <u>empirical</u> or <u>heuristic</u>

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RANSAC = random sample consensus (1981, SRI)

assumption is data consists of "inliers" which fit a model (w/noise) & "outliers" which do not fit the model

goal: find + keep inliers and corresponding model parameters disregarding the outliers

idea:
- choose random subset of your data which uniquely fits model
- count the inliers from all data points, consistent with these parameters
- do this many times, take the one where the number of inliers is maximum, if # of inliers ≈ approx what you expect, then done

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important parameters 9-5

n_0 : # obs. which uniquely fit model

dy : threshold for inlier assignment (30, 45, ...)

k : # trials, iterations

p : probability that ONLY inliers are selected for a random sample of n_0, in any of the k trials (i.e. p = probability that outcome is useful)

w : $\frac{\#\text{ inliers}}{\#\text{ points}}$ (often you estimate)

w^{n_0} : prob. that all n_0 pts. are inliers, for 1 trial (R.S.)

$1 - w^{n_0}$: prob that at least 1 outlier is in the trial

$(1-w^{n_0})^k$: prob that good result never appears in k trials

$(1-w^{n_0})^k = 1-p$

$k \log(1-w^{n_0}) = \log(1-p)$,

$$k = \frac{\log(1-p)}{\log(1-w^{n_0})}$$

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for line fit problem 9-6

if you would do exercise all possible pairs

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n(n-1)(n-2)\cdots(n-k+1)}{(k)(k-1)\cdots(1)}$$

$$\binom{49}{2} = 1176$$

matlab factorial(n) only exact if $n \leq 21$

choose $p = 0.99$

$w = 0.5$

$n_0 = 2$

$$k = \frac{\log(1-p)}{\log(1-w^{n_0})} = 16$$

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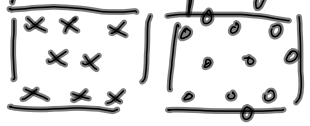
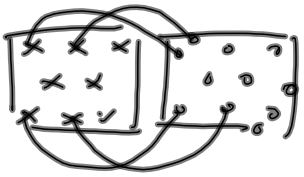
Common applications: photogr. block assembly 9-7

1.  IP, FP, Harris, Förstner SIFT, SURF

2.  match pts to find conjugates

K times:
3. choose $n_0 = 5$ conjugate pts. + compute R/D
4. count inliers

$n = 1000$
$w = 0.75$
$p = 0.99$
$n_0 = 5$
$k = 17$

$p = 0.999$ $k = 25$

$p = 0.9999$ $k = 34$

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