Lecture 12 CE697 Adv. Data Adj. 12-1
Tues. 12 July 2016

next class Thurs. 14th

Schedule

S	M	T	W	Th	F	S
	11	$\boxed{12}$	13	$\boxed{14}$	15	16
17	18	$\boxed{19}$	20	21	22	23
24	25	26	27	28	29	30
31	1	2	3	4	5	6
7	8	9	10	11	12	13

grades due

22nd

Kalman Filter 3 examples:
1 linear tracking target in image
2 NL radar tracking
3 NL attitude estimation

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Seq. LS — adding rows to B matrix 12-2
usual interp.

can also add columns to B matrix
example is adding new obs. on photo of new point

missing:
(1) state vector, new copy @ each epoch
(2) dynamic model (Eq. of Motion)

KF is really just Seq. Linear LS with 3 kinds of eq.
(1) observation
(2) error propagation
(3) prediction equation

NL KF
linearized LKF
extended EKF
iterated extended IEKF
unscented UKF

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12-3

our instinct from NL LS — make taylor series approx, then left with "incremental" (corrections)

$\Delta x, \Delta l, \ldots$ to refine full parameters

X, l

often in practice with NL KF, there are techniques to keep full variables.

good reference: KF for beginners
w/ matlab examples
Phil Kim
AJIN Publ. 2010

ls_kf.pdf LS → conv. eqns for KF

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12-4

epoch 1 $B_1 x_1 = f_1$ obs R

$X_2 = \Phi_1 x_1$ predictions Q

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epoch 2 $B_2 x_2 = f_2$

$x_3 = \phi_2 x_2$

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epoch 3 $B_3 x_3 = f_3$

$x_4 = \phi_3 x_3$

⋮

$$\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}$$

$$\begin{bmatrix} B_1 & & & \\ -\Phi_1 & I & & \\ \hdashline & B_2 & & \\ & -\Phi_2 & I & \\ \hdashline & & B_3 & \\ & & -\Phi_3 & I \\ \hdashline & & & B_4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} f_1 \\ 0 \\ \hdashline f_2 \\ 0 \\ \hdashline f_3 \\ 0 \\ \hdashline f_4 \end{bmatrix}$$

Gilbert Strang

[in theory would do big batch LS solution]

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Linear Case KF algo 12-5

$z_k = H_k x_k + v_k$ (v & $B_0 = \phi$)

$E(v_k v_j^T) = R_k \quad j=k$
$\qquad\qquad 0 \quad j \neq k$

$x_{k+1} = \Phi_k x_k + w_k$

$E(w_k w_j^T) = Q$ if $j=k$
$\qquad\qquad 0 \quad j \neq k$

$\hat{x}^-_{k+1} = \Phi_k \hat{x}_k$

P is cov. matrix for x

$P^-_{k+1} = \Phi_k P_k \Phi_k^T + Q$

$Y = Ax$
$\Sigma_{yy} = A \Sigma_{xx} A^T$

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need to start $\hat{x}^-_0$, P^-_0 initial values 12-6

compute K. gain

$K_k = P^-_k H_k^T (H_k P^-_k H_k^T + R)^{-1}$

update est w/ new meas.

$\hat{x}_k = \hat{x}^-_k + K_k(z_k - H\hat{x}^-_k)$

cov. for updated est.

$P_k = (I - K_k H_k) P^-_k$

project ahead

$\hat{x}^-_{k+1} = \Phi_k \hat{x}_k$

$P^-_{k+1} = \Phi_k P_k \Phi_k^T + Q$

state vector $\begin{bmatrix} x \\ y \end{bmatrix}$ $\begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$ $\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \\ y \\ \dot{y} \\ \ddot{y} \end{bmatrix}$ ... X is state vector

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KF algo for Nonlinear Case (EKF) 12-7

$z_k = h(x_k) + v_k$, $h(x_k)$ is NL , R

$H = \partial h / \partial x$

$x_{k+1} = \varphi(x_k) + w_k$ $\varphi(x)$ is NL Q

$\Phi = \frac{\partial \varphi}{\partial x}$

$Hx \rightarrow h(x)$

$\Phi x \rightarrow \varphi(x)$

Note: revised some things here. will be inconsistent with video. This is better notation. mostly replace g with φ replace G with Φ

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$\hat{x}_0^-$, P_0^- 12-8

comp K. gain

$K_k = P_k^- H^T (H P_k^- H^T + R)^{-1}$ $H = \frac{\partial h}{\partial x}\Big|_{\hat{x}}$

update state vect. est.

$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-))$

error prop

$P_k = P_k^- - K_k H P_k^{-1}$

project ahead

$\hat{x}_{k+1}^- = \varphi(x_k)$ $\boxed{\text{edits here as well}}$

$P_{k+1}^- = \Phi P_k \Phi^T + Q$

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