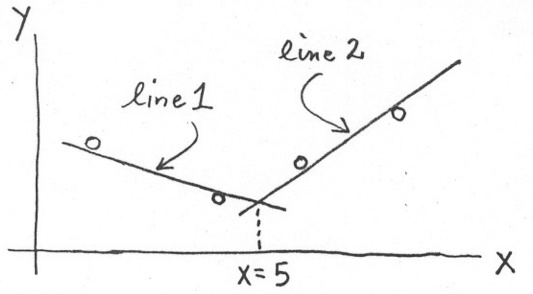
CE 506 Exam I Name ______________________

4 Oct. 2002

1.



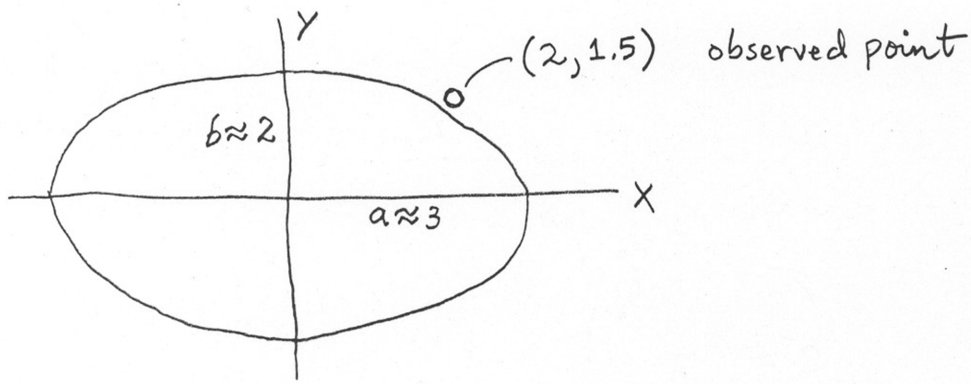

| X | Y | |
|---|---|---|
| 1 | 3.2 | line 1 |
| 4 | 1.4 | |
| 6 | 2.1 | line 2 |
| 8 | 3.9 | |

Two points are observed on each of two straight lines. The lines <u>must</u> <u>intersect</u> at $X = 5$. The y-coordinate is an observation, the x-coordinate is a constant.

(a) show analysis of the problem: $n, n_0, r$

(b) select parameters and show the condition equations in matrix form for the indirect observation method ($v + B\Delta = f$).

2.




The model is an ellipse centered at the origin, of the form $x^2/a^2 + y^2/b^2 = 1$. y-coordinate is an observation, x-coordinate is a constant.

(a) For the indicated observed point, show the linearized condition equation in the form $v + B\Delta = f$ (indirect observation method). Show symbolically and numerically.

(b) If we fit $n$ data points by least squares to this ellipse model by the method of <u>observations</u> <u>only</u>, and we write one condition equation, what is $n$?

# CE 506 Exam I Solution

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1. The unconstrained 2 line problem would have $n_0 = 4$. By adding the constraint that the 2 lines intersect at $x = 5$, we reduce $n_0$ by 1 to $n_0 = 3$. You can show this by,

$$y = m_1 x + b_1 \quad ; \text{ line 1}$$
$$y = m_2 x + b_2 \quad ; \text{ line 2}$$

but the y's must be equal at $x = 5$, therefore

$$m_1 \cdot 5 + b_1 = m_2 \cdot 5 + b_2$$

Solve this for one of the parameters, say $b_1$

$$b_1 = 5m_2 + b_2 - 5m_1$$

then substitute into the original first equation

$$y = m_1 x + (5m_2 + b_2 - 5m_1)$$
$$y = (x-5)m_1 + 5m_2 + b_2$$

now we can describe the 2 constrained lines with 2 equations and $\underline{3}$ parameters. Thus $n_0 = 3$.

$$y = (x-5)m_1 + 5m_2 + b_2 \quad ; \text{ line 1}$$
$$y = m_2 x + b_2 \quad ; \text{ line 2}$$

ok, now write condition equations for observed data,

$$y + v = (x-5)\,m_1 + 5m_2 + b_2$$
$$y + v = x\,m_2 + b_2$$

(a) $n = 4$, $n_0 = 3$, $r = 1$

$$v - (x-5)m_1 - 5m_2 - b_2 = -y$$
$$v - x m_2 - b_2 = -y$$

(b)

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} + \begin{bmatrix} +4 & -5 & -1 \\ +1 & -5 & -1 \\ 0 & -6 & -1 \\ 0 & -8 & -1 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ b_2 \end{bmatrix} = \begin{bmatrix} -3.2 \\ -1.4 \\ -2.1 \\ -3.9 \end{bmatrix}$$

$$v + B \Delta = f$$

2. for the given model and data, $n = 1$, $n_0 = 2$ therefore we do not have enough observations (much less redundancy) to solve anything. But the question just asks us to write one condition equation, so we are ok. For indirect observations we will need $n_0 = 2$ parameters (use $a$ & $b$) and an equation where $y$ is a function of the parameters,

(a)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = b^2 - \frac{b^2}{a^2}x^2 \qquad \text{OR} \qquad y^2 = b^2\left[1 - x^2/a^2\right]$$

$$y = \left[b^2 - \frac{b^2}{a^2}x^2\right]^{1/2} \qquad y = b\sqrt{1 - x^2/a^2}$$

$$\vdots$$

$$F = y - \left[b^2 - \frac{b^2}{a^2}x^2\right]^{1/2} = 0 \qquad \text{etc.}$$

$$B = \begin{bmatrix} \frac{\partial F}{\partial a} & \frac{\partial F}{\partial b} \end{bmatrix}, \quad \text{use } (a^0, b^0) = (3, 2) \text{ as given}$$

$$\frac{\partial F}{\partial a} = -\tfrac{1}{2}[\cdot]^{-1/2}\left(-\left(-2a^{-3}\cdot b^2x^2\right)\right) = \left.\frac{-b^2x^2/a^3}{\sqrt{b^2 - \frac{b^2}{a^2}x^2}}\right|_{a^0=3,\ b^0=2}$$

$$\frac{\partial F}{\partial b} = -\tfrac{1}{2}[\cdot]^{-1/2}\left(2b - \frac{2bx^2}{a^2}\right) = \left.\frac{-b + \frac{bx^2}{a^2}}{\sqrt{b^2 - \frac{b^2}{a^2}x^2}}\right|_{a^0=3,\ b^0=2}$$

$$f = -F = -y + \left.\left[b^2 - \frac{b^2}{a^2}x^2\right]^{1/2}\right|_{a^0=3,\ b^0=2}$$

point: $(2, 1.5)$

$$v + \begin{bmatrix} -.3975 & -.7454 \end{bmatrix} \begin{bmatrix} \Delta a \\ \Delta b \end{bmatrix} = -.0093$$

(b) for observations only, $c = r$

$$\left.\begin{array}{c} n \\ -n_0 \\ \hline r \end{array} \quad \begin{array}{c} n \\ -2 \\ \hline 1 \end{array}\right\} \Rightarrow \underline{n \text{ must be } 3}$$