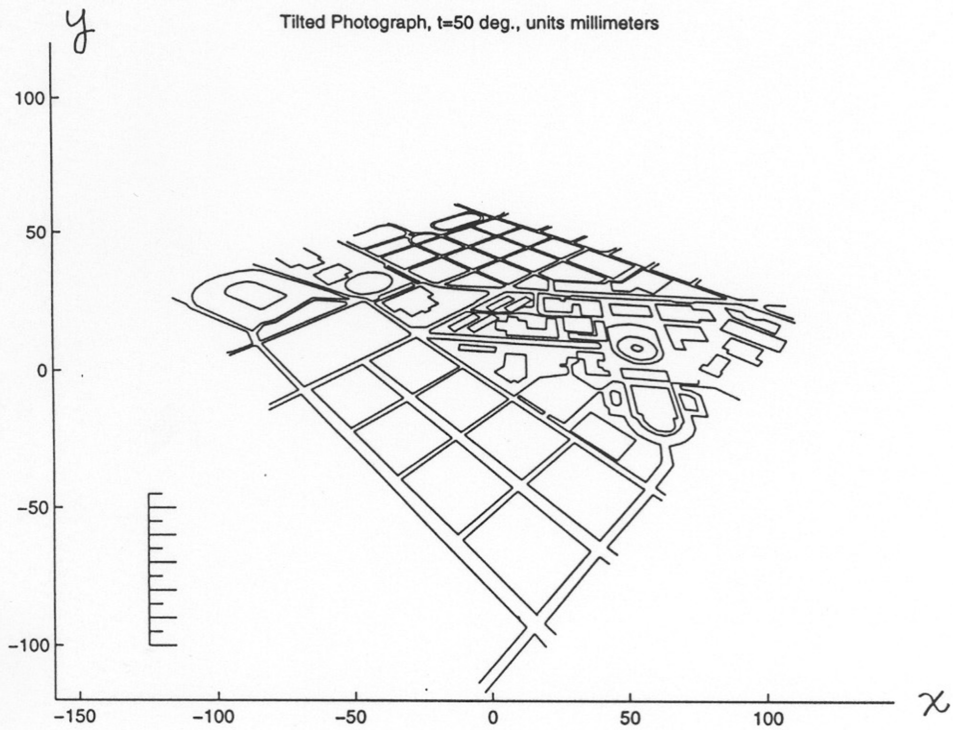
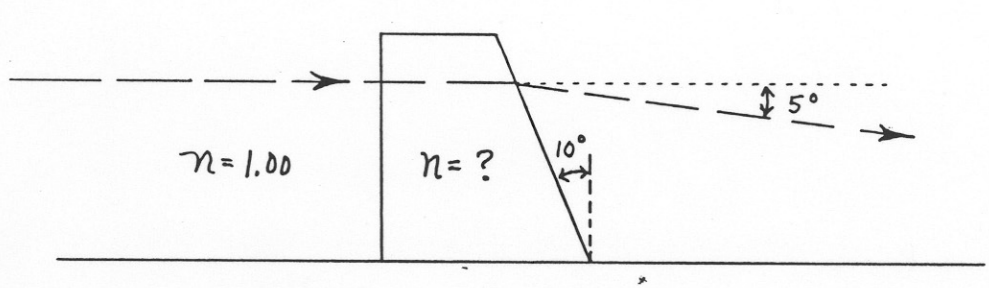
24 Oct '01

CE 503 Exam I Name ____________

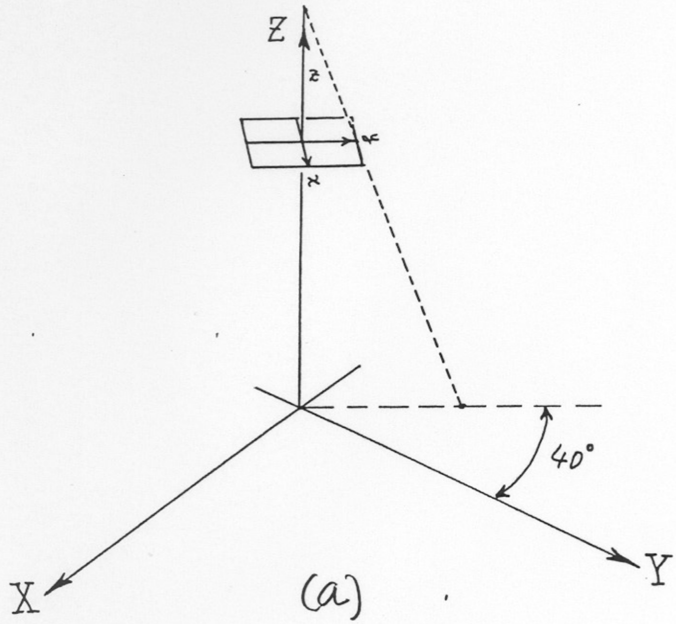
1. In the simulated photograph locate Mackey Arena. Assume a diameter of 98.5 m. Measure image dimensions to compute an X-scale and a Y-scale. Then use the formulae to compute these and show they are consistent. $S_x = \frac{e \sin t}{H}$, $S_y = \frac{e^2 \sin^2 t}{Hf}$. The focal length = 150mm, the tilt = 50°, the camera height above the ground is 430m. $e = \frac{f}{\tan t} - y$. Transfer the graphical scale to measure image.

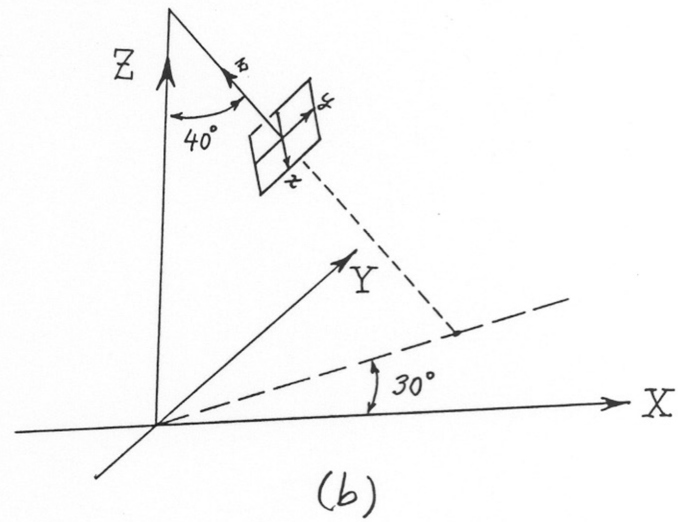


2. The prism in the sketch has a vertical front face, and a rear face tilted at 10°. If we want an incoming horizontal ray to emerge aimed <u>down</u> 5° from the horizontal, what must the <u>index of refraction of the glass</u> be ?



3. For each of the sketches (a) and (b) find a <u>set of sequential rotations</u> that take XYZ into xyz. You need to specify only the type of elementary matrix M_x, M_y, M_z; the order; and the sign and magnitude of each angle.





4. The rotation matrix associated with a frame photograph is

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .1736 & .9848 \\ 0 & -.9848 & .1736 \end{bmatrix}$$

The focal length is 50mm. The location of the perspective center in object space is

$$\begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} m$$

For an image point at $(x, y) = (15, 10)$ mm, what is the corresponding <u>object point</u>, if we fix Y at 20m ?

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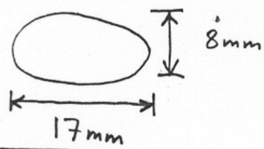
(1.) Scale at Mackey arena

(a) by measurement:



Actual diameter of building = 98.5 m

$$S_x = \frac{17\ \text{mm}}{98{,}500\ \text{mm}} = \boxed{\frac{1}{5794}}\ ,\quad S_y = \frac{8\ \text{mm}}{98500} = \boxed{\frac{1}{12{,}300}}$$

(b) by image geometry:

$H = 430\,m$

$f = 150\,mm$

tilt $t = 50°$

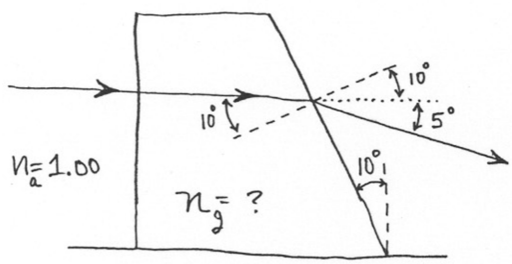
position in image $(x,y) = (-44, +30)\,mm$

$$e = \frac{f}{\tan t} - y = 95.865\ mm$$

$$S_x = \frac{e \cdot \sin t}{H} = \boxed{\frac{1}{5855}}$$

$$S_y = \frac{e^2 \sin^2 t}{H f} = \boxed{\frac{1}{11{,}960}}$$

(2.)



$$n_g \sin 10° = n_a \sin 15°$$

$$n_g = n_a \frac{\sin 15°}{\sin 10°} = 1.00\ \frac{.25882}{.17365}$$

$$\boxed{n_g = 1.490}$$

(3.) (a) rotation only about Z axis

$M_Z(+40°)$

(b) first rotation about Z axis $-60°$

second rotation about X' axis $+40°$

$$M = M_X(+40°)\, M_Z(-60°)$$

(4.)
$$\begin{bmatrix} x \\ y \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix},\quad \begin{bmatrix} X_L \\ Y_L \\ Z_L \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},\quad \frac{1}{\lambda} M^T \begin{bmatrix} x \\ y \\ -f \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$M^T \begin{bmatrix} x \\ y \\ -f \end{bmatrix} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .1736 & -.9848 \\ 0 & .9848 & .1736 \end{bmatrix} \begin{bmatrix} 15 \\ 10 \\ -50 \end{bmatrix} = \begin{bmatrix} 15 \\ 50.976 \\ 1.168 \end{bmatrix}$$

, now fix Y = 20

$$\frac{1}{\lambda} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} X \\ 20 \\ Z \end{bmatrix}$$

divide first and third equations by the second one

$$\frac{u}{v} = \frac{X}{20}\ ,\quad \frac{w}{v} = \frac{Z}{20}$$

$$X = 20\,\frac{15}{50.976} = 5.885\ ,\quad Z = 20\,\frac{1.168}{50.976} = 0.458\ ,\quad \boxed{\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 5.885 \\ 20.000 \\ 0.458 \end{bmatrix}}$$