CE 503 Exam 2 Name ____________

12 Dec 2002

1. The 7-parameter transformation, shown here, is used to relate points in 2 coordinate systems, differing by a scale, 3 rotations, and 3 shifts:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda \mathbf{M} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \quad ; \quad M = M_\kappa M_\varphi M_\omega$$

(a) derive a pseudo-linear, over-parameterized version of this model (like the DLT).

(b) then show how you could use the linear parameters to recover the real, physical parameters $\lambda, \omega, \varphi, \kappa, t_x, t_y, t_z$.

(c) Under what conditions would this give good results, and when would it fail?

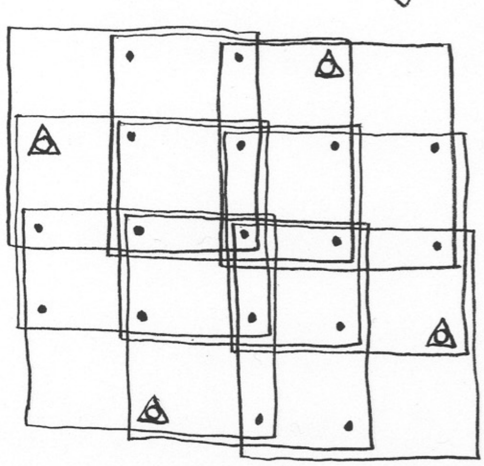
$$\mathbf{M} = \begin{bmatrix} \cos\phi\cos\kappa & \cos\omega\sin\kappa + \sin\omega\sin\phi\cos\kappa & \sin\omega\sin\kappa - \cos\omega\sin\phi\cos\kappa \\ -\cos\phi\sin\kappa & \cos\omega\cos\kappa - \sin\omega\sin\phi\sin\kappa & \sin\omega\cos\kappa + \cos\omega\sin\phi\sin\kappa \\ \sin\phi & -\sin\omega\cos\phi & \cos\omega\cos\phi \end{bmatrix}$$

2. In the following block triangulation, to be solved by collinearity equations, what are

(a) number of equations (n) ?

(b) number of unknowns (n_o) ?

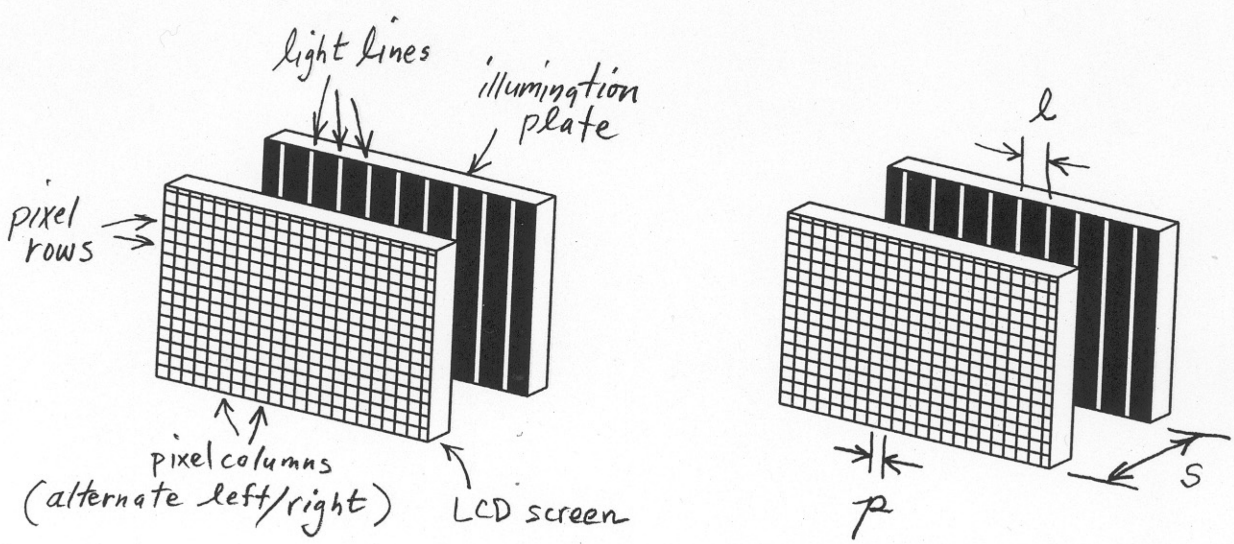
(c) redundancy ($r = n - n_o$) ?

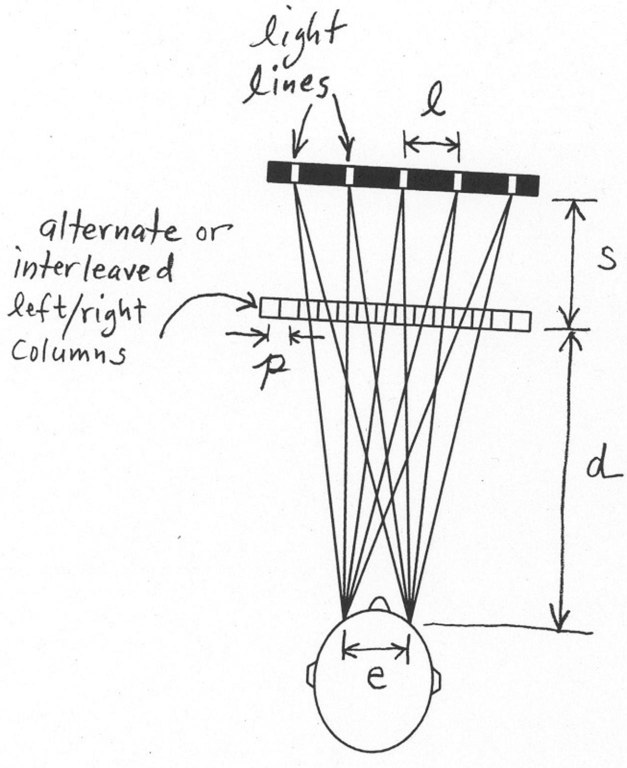


9-photo block

- • = pass point
- ○ = vertical control point (Z)
- △ = horizontal control point (XY)
- △ with ○ inside = full control point (XYZ)

3.





$e = 65$ mm

$p = 0.28$ mm

$d = 610$ mm

You must design an "auto stereo" viewer similar to the DTI monitor shown in class.

e = eyebase

l = pitch (spacing) of light lines

s = separation between illumination plate and LCD screen

p = screen pixel pitch (spacing)

d = distance from eye to screen

For the given e, p, and d, what are l and s?

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1.
(a)
$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda M \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \rightarrow \begin{array}{l} X = \overset{L_1}{\lambda m_{11}} x + \overset{L_2}{\lambda m_{12}} y + \overset{L_3}{\lambda m_{13}} z + \overset{L_{10}}{t_x} \\ Y = \overset{L_4}{\lambda m_{21}} x + \overset{L_5}{\lambda m_{22}} y + \overset{L_6}{\lambda m_{23}} z + \overset{L_{11}}{t_y} \\ Z = \overset{L_7}{\lambda m_{31}} x + \overset{L_8}{\lambda m_{32}} y + \overset{L_9}{\lambda m_{33}} z + \overset{L_{12}}{t_z} \end{array}$$

with the indicated substitution, we write 3 linear equations per point:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x & y & z & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & x & y & z & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & x & y & z & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \\ L_3 \\ L_4 \\ L_5 \\ L_6 \\ L_7 \\ L_8 \\ L_9 \\ L_{10} \\ L_{11} \\ L_{12} \end{bmatrix}$$

; with minimum of 4 points, you solve linearly for $L_1 \rightarrow L_{12}$ then extract the physical parameters as follows:

(b)

$t_x = L_{10}$
$t_y = L_{11}$
$t_z = L_{12}$

$\lambda = [L_1^2 + L_2^2 + L_3^2]^{1/2}$
$m_{11} = L_1/\lambda$
$m_{12} = L_2/\lambda$
$m_{13} = L_3/\lambda$

$m_{21} = L_4/\lambda$
$m_{22} = L_5/\lambda$
$m_{23} = L_6/\lambda$

$m_{31} = L_7/\lambda$
$m_{32} = L_8/\lambda$
$L_{33} = L_9/\lambda$

$\varphi = \sin^{-1}(m_{31})$
$\omega = \tan^{-1}\left(\dfrac{-m_{32}/\cos\varphi}{m_{33}/\cos\varphi}\right)$
$\kappa = \tan^{-1}\left(\dfrac{-m_{21}/\cos\varphi}{m_{11}/\cos\varphi}\right)$

(c) This approach should yield good results when
- the data is well distributed, and
- the measurement errors are very small

It will not yield good results when either of the above conditions are not met

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2. 9-photo block adjustment, counting equations and unknown parameters

first tabulate number of rays (photos) per point for each point in the 5 rows of points:

```
     2  3  2
  2  4  6  4  2
  3  6  9  6  3
  2  4  6  4  2
     2  3  2
```

adding all of these ⇒ 77 rays
⇒ 154 equations
(2 equations/ray)

now for the unknowns, each photo contributes 6 unknowns $\omega, \varphi, \kappa, X_L, Y_L, Z_L$

$9 \times 6 = 54$ unknowns for photos

each point contributes 3 unknowns X, Y, Z (except the control points which do not contribute any unknowns, $(21-4) \times 3 = 51$ unknowns for points

number of equations	(n)	154
number of unknowns	(n_o)	54+51 = 105
redundancy	(r)	49

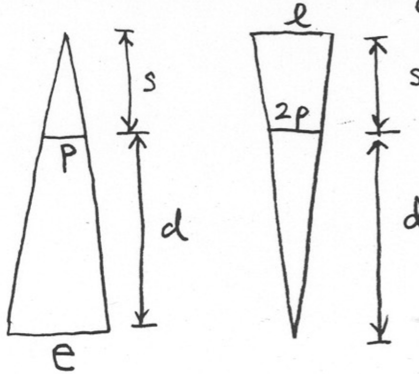
(note: assumption that x_o, y_o, and f are known, fixed constants.)

3. "DTI" Screen design:

$e = 65$ mm
$p = .28$ mm
$d = 610$ mm

given

note there are 2 triangles to consider:



from first triangle $\frac{p}{e} = \frac{s}{s+d}$

from second triangle $\frac{2p}{l} = \frac{d}{s+d}$

2 equations, 2 unknowns l, s

first equation

$$\frac{e}{p} = \frac{s+d}{s} = 1 + \frac{d}{s} \; , \quad \frac{e}{p} - 1 = \frac{d}{s} \; , \quad \boxed{s = \frac{d}{e/p - 1}}$$

$\underline{s = 2.6391 \text{ mm}}$

second equation

$$\frac{l}{2p} = \frac{s+d}{d} = \frac{s}{d} + 1 \; , \quad \boxed{l = 2p\left(s/d + 1\right)}$$

$\underline{l = 0.5624 \text{ mm}}$