CE 506 Exam II   14 Nov, 2002

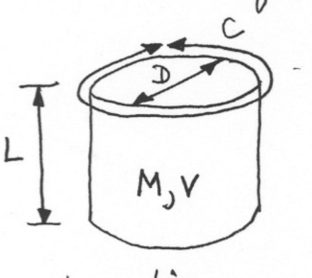
______________________ Name

1. Show the condition equations in matrix form - necessary to fit a plane through the given 4 points, using the method of <u>indirect observations</u>. The z is observed, the x and y are constants. The functional model is $z = a_0 + a_1 x + a_2 y$. Show in the form $V + B\Delta = f$ (DO NOT SOLVE).

$(x, y, z) \rightarrow$ (1, 4, 1.20)   (2, 4, 1.35)

(1, 3, 1.00)   (2, 3, 1.25)

2. The model is the size, shape, and mass of a cylindrical object, with <u>fixed</u> density, S (mass/volume). You observe the length, L, the diameter, D, the circumference, C, the mass, M, and the volume, V. If you were to adjust these observations by the method of least squares, <u>observations only</u>, show the condition equations in the form $Av = f$.



observations

3. You have made an adjustment where $n_0 = 2$, and the stochastic model is $\sigma_0^2 = 4$, $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 2$, $\sigma_4^2 = \sigma_5^2 = 4$ (a priori values). The adjustment produces a residual vector as shown. Make the "global" hypothesis test on the reference variance, at $\alpha = 0.10$ significance level.

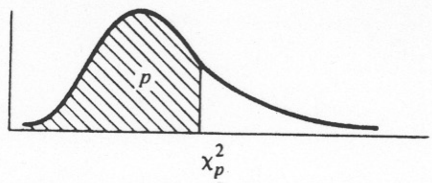
$H_0 : \sigma^2 = \sigma_0^2$

vs.

$H_1 : \sigma^2 > \sigma_0^2$

$$V = \begin{bmatrix} 1.6 \\ -1.8 \\ 2.0 \\ 2.8 \\ -2.7 \end{bmatrix}$$

Table II. Percentiles of the Chi-Square Distribution



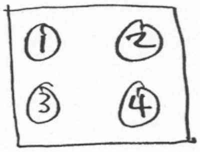
DEGREES OF FREEDOM	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.20}$	$\chi^2_{.30}$
1	.000	.000	.001	.004	.016	.064	.148
2	.010	.020	.051	.103	.211	.446	.713
3	.072	.115	.216	.352	.584	1.00	1.42
4	.207	.297	.484	.711	1.06	1.65	2.20
5	.412	.554	.831	1.15	1.61	2.34	3.00
6	.676	.872	1.24	1.64	2.20	3.07	3.83
7	.989	1.24	1.69	2.17	2.83	3.82	4.67
8	1.34	1.65	2.18	2.73	3.49	4.59	5.53
9	1.73	2.09	2.70	3.33	4.17	5.38	6.39
10	2.16	2.56	3.25	3.94	4.87	6.18	7.27
11	2.60	3.05	3.82	4.57	5.58	6.99	8.15
12	3.07	3.57	4.40	5.23	6.30	7.81	9.03
13	3.57	4.11	5.01	5.89	7.04	8.63	9.93
14	4.07	4.66	5.63	6.57	7.79	9.47	10.8
15	4.60	5.23	6.26	7.26	8.55	10.3	11.7
16	5.14	5.81	6.91	7.96	9.31	11.2	12.6
17	5.70	6.41	7.56	8.67	10.1	12.0	13.5
18	6.26	7.01	8.23	9.39	10.9	12.9	14.4
19	6.83	7.63	8.91	10.1	11.7	13.7	15.4
20	7.43	8.26	9.59	10.9	12.4	14.6	16.3
21	8.03	8.90	10.3	11.6	13.2	15.4	17.2
22	8.64	9.54	11.0	12.3	14.0	16.3	18.1
23	9.26	10.2	11.7	13.1	14.8	17.2	19.0
24	9.89	10.9	12.4	13.8	15.7	18.1	19.9
25	10.5	11.5	13.1	14.6	16.5	18.9	20.9
26	11.2	12.2	13.8	15.4	17.3	19.8	21.8
27	11.8	12.9	14.6	16.2	18.1	20.7	22.7
28	12.5	13.6	15.3	16.9	18.9	21.6	23.6
29	13.1	14.3	16.0	17.7	19.8	22.5	24.6
30	13.8	15.0	16.8	18.5	20.6	23.4	25.5
40	20.7	22.1	24.4	26.5	29.0	32.3	34.9
50	28.0	29.7	32.3	34.8	37.7	41.4	44.3
60	35.5	37.5	40.5	43.2	46.5	50.6	53.8

$\chi^2_{.50}$	$\chi^2_{.70}$	$\chi^2_{.80}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$
.455	1.07	1.64	2.71	3.84	5.02	6.63	7.88
1.39	2.41	3.22	4.61	5.99	7.38	9.21	10.6
2.37	3.66	4.64	6.25	7.81	9.35	11.3	12.8
3.36	4.88	5.99	7.78	9.49	11.1	13.3	14.9
4.35	6.06	7.29	9.24	11.1	12.8	15.1	16.7
5.35	7.23	8.56	10.6	12.6	14.4	16.8	18.5
6.35	8.38	9.80	12.0	14.1	16.0	18.5	20.3
7.34	9.52	11.0	13.4	15.5	17.5	20.1	22.0
8.34	10.7	12.2	14.7	16.9	19.0	21.7	23.6
9.34	11.8	13.4	16.0	18.3	20.5	23.2	25.2
10.3	12.9	14.6	17.3	19.7	21.9	24.7	26.8
11.3	14.0	15.8	18.5	21.0	23.3	26.2	28.3
12.3	15.1	17.0	19.8	22.4	24.7	27.7	29.8
13.3	16.2	18.2	21.1	23.7	26.1	29.1	31.3
14.3	17.3	19.3	22.3	25.0	27.5	30.6	32.8
15.3	18.4	20.5	23.5	26.3	28.8	32.0	34.3
16.3	19.5	21.6	24.8	27.6	30.2	33.4	35.7
17.3	20.6	22.8	26.0	28.9	31.5	34.8	37.2
18.3	21.7	23.9	27.2	30.1	32.9	36.2	38.6
19.3	22.8	25.0	28.4	31.4	34.2	37.6	40.0
20.3	23.9	26.2	29.6	32.7	35.5	38.9	41.4
21.3	24.9	27.3	30.8	33.9	36.8	40.3	42.8
22.3	26.0	28.4	32.0	35.2	38.1	41.6	44.2
23.3	27.1	29.6	33.2	36.4	39.4	43.0	45.6
24.3	28.2	30.7	34.4	37.7	40.6	44.3	46.9
25.3	29.2	31.8	35.6	38.9	41.9	45.6	48.3
26.3	30.3	32.9	36.7	40.1	43.2	47.0	49.6
27.3	31.4	34.0	37.9	41.3	44.5	48.3	51.0
28.3	32.5	35.1	39.1	42.6	45.7	49.6	52.3
29.3	33.5	36.2	40.3	43.8	47.0	50.9	53.7
39.3	44.2	47.3	51.8	55.8	59.3	63.7	66.8
49.3	54.7	58.2	63.2	67.5	71.4	76.2	79.5
59.3	65.2	69.0	74.4	79.1	83.3	88.4	92.0

CE 506 Exam II Solution 19 November 2002

1. fit 4 points to a plane, Z observation, xy constant

$n=4$
$n_0=3$
$r=1$



$$Z = a_0 + a_1 x + a_2 y$$

$$F = Z - a_0 - a_1 x - a_2 y = 0$$

$$v_z - a_0 - a_1 x - a_2 y = -Z$$

$$\begin{bmatrix} v_{z_1} \\ v_{z_2} \\ v_{z_3} \\ v_{z_4} \end{bmatrix} + \begin{bmatrix} -1 & -1 & -4 \\ -1 & -2 & -4 \\ -1 & -1 & -3 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -1.20 \\ -1.35 \\ -1.00 \\ -1.25 \end{bmatrix}$$

$$v + B\Delta = f$$

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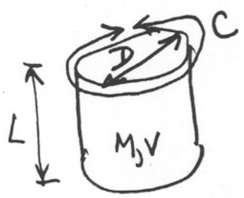
2. size, shape, and mass of cylindrical object with fixed density S, observe L, D, C, M, V



$n=5$
$n_0=2$ (L, D)
$r=3$

3 cond. eqn:
1. $\pi D = C$
2. $V = \pi \left(\frac{D}{2}\right)^2 \cdot L$
3. $M/V = S$ or $M = VS$

note: the D^2 makes it nonlinear

$F_1 = \pi D - C = 0$

$F_2 = \frac{\pi D^2 L}{4} - V = 0$

$F_3 = M - VS = 0$

$$\begin{array}{ccccc} L & D & C & M & V \end{array}$$

$$\underbrace{\begin{bmatrix} 0 & \pi & -1 & 0 & 0 \\ \frac{\pi D^2}{4} & \frac{\pi D L}{2} & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & -S \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_L \\ v_D \\ v_C \\ v_M \\ v_V \end{bmatrix}}_{v} = \underbrace{-\begin{bmatrix} \pi D - C \\ \frac{\pi D^2 L}{4} - V \\ M - VS \end{bmatrix} - A(\ell - \ell^0)}_{f}$$

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3. $n_0 = 2$, $n = 5 \Rightarrow r = 3$

$H_0 : \sigma^2 = \sigma_0^2$

vs.

$H_1 : \sigma^2 > \sigma_0^2$

$$V = \begin{bmatrix} 1.6 \\ -1.8 \\ 2.0 \\ 2.8 \\ -2.7 \end{bmatrix}$$

$$\sigma_0^2 = 4, \quad W = \begin{bmatrix} 4/2 & & & & \\ & 4/2 & & & \\ & & 4/2 & & \\ & & & 4/4 & \\ & & & & 4/4 \end{bmatrix} = \begin{bmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & 1 & \\ & & & & 1 \end{bmatrix}$$

test statistic $= q = \frac{r \cdot \hat{\sigma}_0^2}{\sigma_0^2} = \frac{v^T W v}{\sigma_0^2} \sim \chi^2_r$, make test at $\alpha = 0.10$ significance level

Decision Rule: Reject Ho if $q > \chi^2_{1-\alpha, 3}$

α: probability of Type I error (reject Ho when true)

$$\left.\begin{array}{l} q = \frac{v^T W v}{\sigma_0^2} = \frac{34.73}{4} = 8.68 \\ \\ \chi^2_{.90,3} = 6.25 \end{array}\right\} \begin{array}{l} q > \chi^2_{.90,3} \text{ so } \boxed{\text{reject Ho}} \leftarrow \\ 8.68 > 6.25 \end{array}$$