CE 506 Exam III 9 Dec 2002

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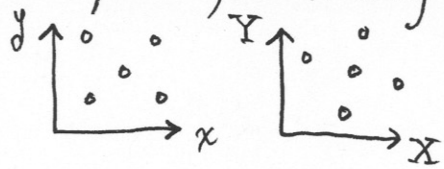
Name

1. General Least Squares

(a) for each case, what are n, n_0, r ?

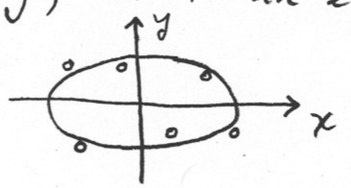
(i) 5 points observed in both systems, related by 4 parameter transformation?

$$x = aX + bY + t_x$$
$$y = -bX + aY + t_y$$



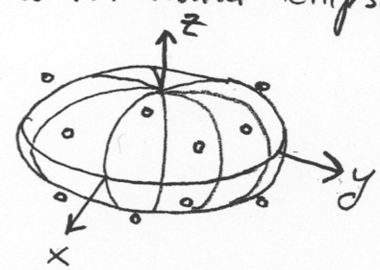
(ii) 6 points observed in x and y, fit to an ellipse?

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$



(iii) 10 points observed in $x, y,$ and z fit to a rotational ellipsoid?

$$\frac{(x-x_0)^2 + (y-y_0)^2}{a^2} + \frac{(z-z_0)^2}{b^2} = 1$$



(b) for case (i) above (4-parameter transformation) we have intermediate results (after 1 iteration for example) for a single point

original observations	
x	11.1
y	14.0
X	3.0
Y	3.8

current, updated observations	
x^0	11.0
y^0	13.9
X^0	3.1
Y^0	3.9

current, updated parameters	
a^0	2.0
b^0	0.0
t_x^0	5.0
t_y^0	6.0

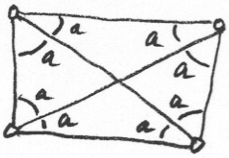
(i) show, in matrix form, the linearized condition equations for this point.

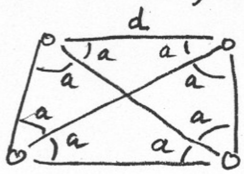
(ii) after solving the current iteration, we obtain: show the linearized condition equations for this point for the next iteration

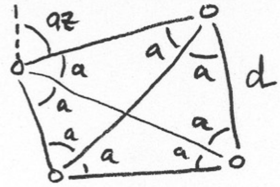
$\Delta a = 0.1$	$v_x = 0.5$
$\Delta b = 0.1$	$v_y = -0.5$
$\Delta t_x = 0.1$	$v_X = -0.1$
$\Delta t_y = 0.1$	$v_Y = -0.1$

2. 2D Network Adjustment

(a) what is the minimum control point information needed for each network? (partial/fractional control is allowed, i.e. only X or only Y).

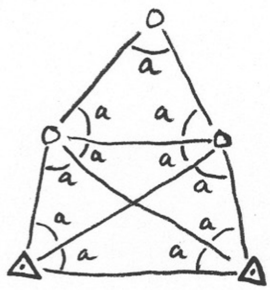
(i) 

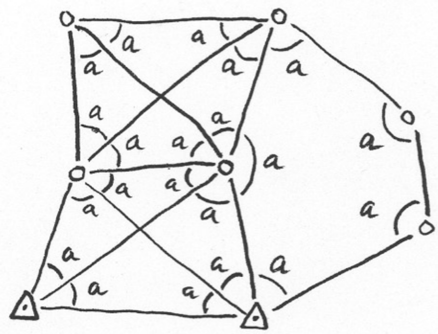
(ii) 

(iii) 

a = angle observation, az = azimuth observation, d = distance observation

(b) we wish to solve the networks for coordinates of the unknown points (pt) by indirect observations. (△ = control point, known XY) in each case, what is the redundancy? (a = angle observation)

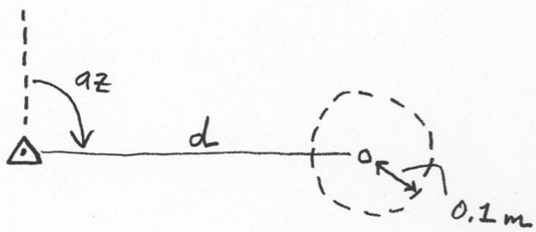
(i) 

(ii) 

(c) the uncertainty (σ) in the unknown point determination is

- 0.1m from the distance observation, and
- 0.1m from the azimuth observation

(each considered separately)



d is approximately 200m

a priori $\sigma_0^2 = 1$

what are the weights for the

(i) distance observation?

(ii) azimuth observation?

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– Solution –

1 (a) <u>General Least Squares</u>

(i) 5 points observed x_i, y_i, X_i, Y_i

$$x = aX + bY + t_x$$
$$y = -bX + aY + t_y$$

$n = 20$
$n_0 = 4 + (5 \times 2) = 14$
$r = 6$

(ii) 6 points observed in x, y to fit ellipse

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

$n = 12$
$n_0 = 4 + 6 = 10$
$r = 2$

(iii) 10 points observed in x, y, z to fit rotational ellipsoid

$$\frac{(x-x_0)^2 + (y-y_0)^2}{a^2} + \frac{(z-z_0)^2}{b^2} = 1$$

$n = 30$
$n_0 = 5 + (10 \times 2) = 25$
$r = 5$

(b) evaluate condition equation for 1 point of (i) above, where

$x = 11.1$	$x^\circ = 11.0$	$a^\circ = 2.0$	
$y = 14.0$	$y^\circ = 13.9$	$b^\circ = 0.0$	
$X = 3.0$	$X^\circ = 3.1$	$t_x^\circ = 5.0$	
$Y = 3.8$	$Y^\circ = 3.9$	$t_y^\circ = 6.0$	
(original obs)	(updated obs)	(current parameters)	

$$\boxed{\begin{aligned} F_x &= x - aX - bY - t_x = 0 \\ F_y &= y + bX - aY - t_y = 0 \end{aligned}}$$

(i) $A = \left.\partial F/\partial \ell\right|_{\ell^\circ, x^\circ}$ $\quad B = \left.\frac{\partial F}{\partial x}\right|_{\ell^\circ, x^\circ}$ $\quad f = -F^\circ_{\ell^\circ, x^\circ} - A(\ell - \ell^\circ)$

$$A = \begin{array}{c} \begin{matrix} (x) & (y) & (X) & (Y) \end{matrix} \\ \begin{bmatrix} 1 & 0 & -a^\circ & -b^\circ \\ 0 & 1 & b^\circ & -a^\circ \end{bmatrix} \end{array}, \quad B = \begin{array}{c} \begin{matrix} (a) & (b) & (t_x) & (t_y) \end{matrix} \\ \begin{bmatrix} -X^\circ & -Y^\circ & -1 & 0 \\ -Y^\circ & X^\circ & 0 & -1 \end{bmatrix} \end{array}$$

$$f = -\begin{bmatrix} (x^\circ - a^\circ X^\circ - b^\circ Y^\circ - t_x^\circ) \\ (y^\circ + b^\circ X^\circ - a^\circ Y^\circ - t_y^\circ) \end{bmatrix} - \begin{pmatrix} 1 & 0 & -a^\circ & -b^\circ \\ 0 & 1 & b^\circ & -a^\circ \end{pmatrix} \left(\begin{bmatrix} x \\ y \\ X \\ Y \end{bmatrix} - \begin{bmatrix} x^\circ \\ y^\circ \\ X^\circ \\ Y^\circ \end{bmatrix} \right)$$

evaluating with numbers, $A = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} -3.1 & -3.9 & -1.0 & 0.0 \\ -3.9 & 3.1 & 0.0 & -1.0 \end{bmatrix}$

$$f = -\begin{pmatrix} -0.2 \\ 0.1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix} \left(\begin{bmatrix} 11.1 \\ 14.0 \\ 3.0 \\ 3.8 \end{bmatrix} - \begin{bmatrix} 11.0 \\ 13.9 \\ 3.1 \\ 3.9 \end{bmatrix} \right) = \begin{pmatrix} 0.2 \\ -0.1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} 0.1 \\ 0.1 \\ -0.1 \\ -0.1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ -0.1 \end{pmatrix} - \begin{pmatrix} 0.3 \\ 0.3 \end{pmatrix} = \begin{pmatrix} -0.1 \\ -0.4 \end{pmatrix}$$

current iteration:

$$\boxed{\underline{Av + B\Delta = f} : \underbrace{\begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_x \\ v_y \\ v_X \\ v_Y \end{bmatrix}}_{v} + \underbrace{\begin{bmatrix} -3.1 & -3.9 & -1.0 & 0.0 \\ -3.9 & 3.1 & 0.0 & -1.0 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} \Delta a \\ \Delta b \\ \Delta t_x \\ \Delta t_y \end{bmatrix}}_{\Delta} = \underbrace{\begin{bmatrix} -0.1 \\ -0.4 \end{bmatrix}}_{f}}$$

1 (b) (ii) update for next iteration, call parameter vector P

$$P_{new} = P_{current} + \Delta \,, \quad \begin{bmatrix} 2.0 \\ 0.0 \\ 5.0 \\ 6.0 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 2.1 \\ 0.1 \\ 5.1 \\ 6.1 \end{bmatrix} \xrightarrow{\text{new}} \begin{bmatrix} a^\circ \\ b^\circ \\ t_x^\circ \\ t_y^\circ \end{bmatrix}$$

to update observations we add residuals to original observations

$$\begin{bmatrix} 11.1 \\ 14.0 \\ 3.0 \\ 3.8 \end{bmatrix} + \begin{bmatrix} 0.5 \\ -0.5 \\ -0.1 \\ -0.1 \end{bmatrix} = \begin{bmatrix} 11.6 \\ 13.5 \\ 2.9 \\ 3.7 \end{bmatrix} \xrightarrow{\text{new}} \begin{bmatrix} x^\circ \\ y^\circ \\ X^\circ \\ Y^\circ \end{bmatrix}, \quad \text{as before } A = \begin{bmatrix} 1 & 0 & -2.1 & -0.1 \\ 0 & 1 & 0.1 & -2.1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2.9 & -3.7 & -1 & 0 \\ -3.7 & 2.9 & 0 & -1 \end{bmatrix}, \quad f = -\begin{pmatrix} 0.04 \\ -0.08 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2.1 & -0.1 \\ 0 & 1 & 0.1 & -2.1 \end{pmatrix} \left[\begin{pmatrix} 11.1 \\ 14.0 \\ 3.0 \\ 3.8 \end{pmatrix} - \begin{pmatrix} 11.6 \\ 13.5 \\ 2.9 \\ 3.7 \end{pmatrix} \right]$$

$$f = \begin{pmatrix} -0.04 \\ 0.08 \end{pmatrix} - \begin{pmatrix} 1 & 0 & -2.1 & -0.1 \\ 0 & 1 & 0.1 & -2.1 \end{pmatrix} \begin{pmatrix} -0.5 \\ 0.5 \\ 0.1 \\ 0.1 \end{pmatrix} = \begin{pmatrix} -0.04 \\ 0.08 \end{pmatrix} - \begin{pmatrix} -0.72 \\ 0.3 \end{pmatrix} = \begin{pmatrix} 0.68 \\ -0.22 \end{pmatrix}$$

next iteration:

$$\boxed{\underline{Av + B\Delta = f} : \quad \underbrace{\begin{bmatrix} 1 & 0 & -2.1 & -0.1 \\ 0 & 1 & 0.1 & -2.1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} v_x \\ v_y \\ v_X \\ v_Y \end{bmatrix}}_{v} + \underbrace{\begin{pmatrix} -2.9 & -3.7 & -1 & 0 \\ -3.7 & 2.9 & 0 & -1 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} \Delta a \\ \Delta b \\ \Delta t_x \\ \Delta t_y \end{pmatrix}}_{\Delta} = \underbrace{\begin{pmatrix} 0.68 \\ -0.22 \end{pmatrix}}_{f}}$$

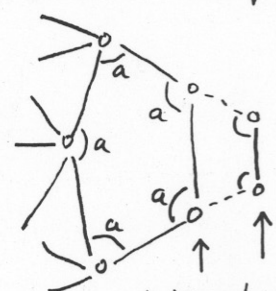
2. (a) (i) two full control points.

(ii) one full control point and one coordinate of a second control point

(iii) one full control point

(b) (i) $n = 11$
$n_0 = 6$
$r = 5$

(ii) the given observations DO NOT DETERMINE the model so there IS NO REDUNDANCY.



both sets of points satisfy the angle observations

(c) $\sigma_D = 0.1\,m$

$\sigma_{AZ} = \tan^{-1}(0.1/200) = .0286 \text{ deg} = .0005 \text{ Rad}$ $\qquad \sigma_0^2 = 1$

(i) $W_D = \dfrac{1}{(0.1)^2} = 100$

(ii) $W_{AZ} = \dfrac{1}{(0.0005)^2} = 4\,000\,000$