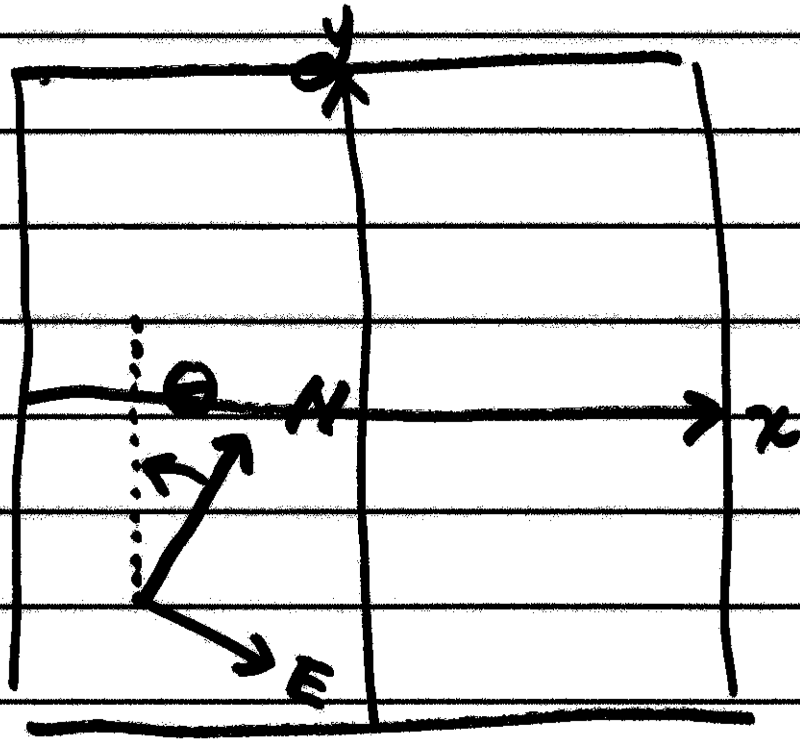


$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} E \\ N \\ U \end{pmatrix}$$

$$\theta_z = K = +30°$$

$$M = M_K = \begin{bmatrix} \cos K & \sin K & 0 \\ -\sin K & \cos K & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

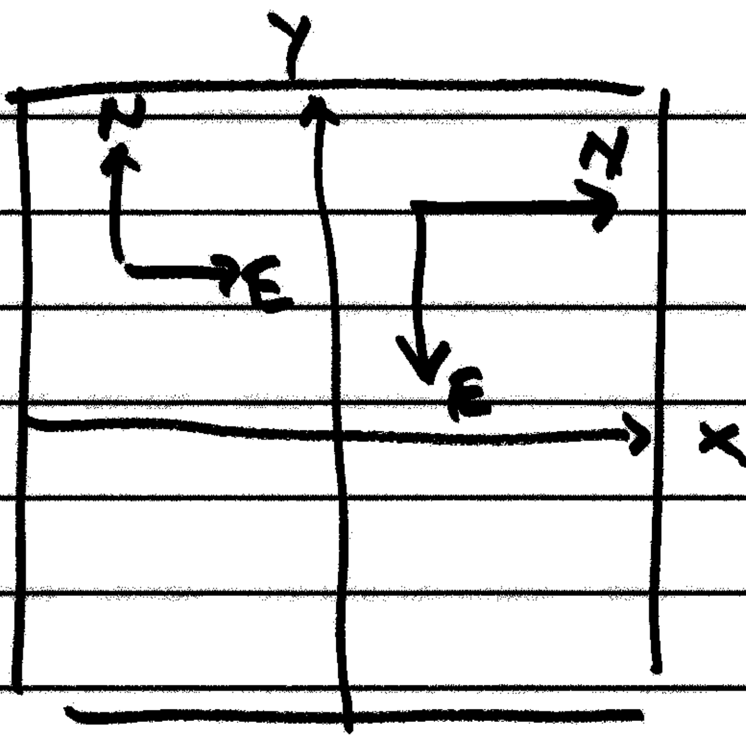
$$M^T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} E \\ N \\ U \end{pmatrix}$$

orthogonal matrix

- inner product (dot product) of any row with itself = 1
  (any column with itself = 1)

- inner product of any row with any other row = 0
  any column with any other column = 0

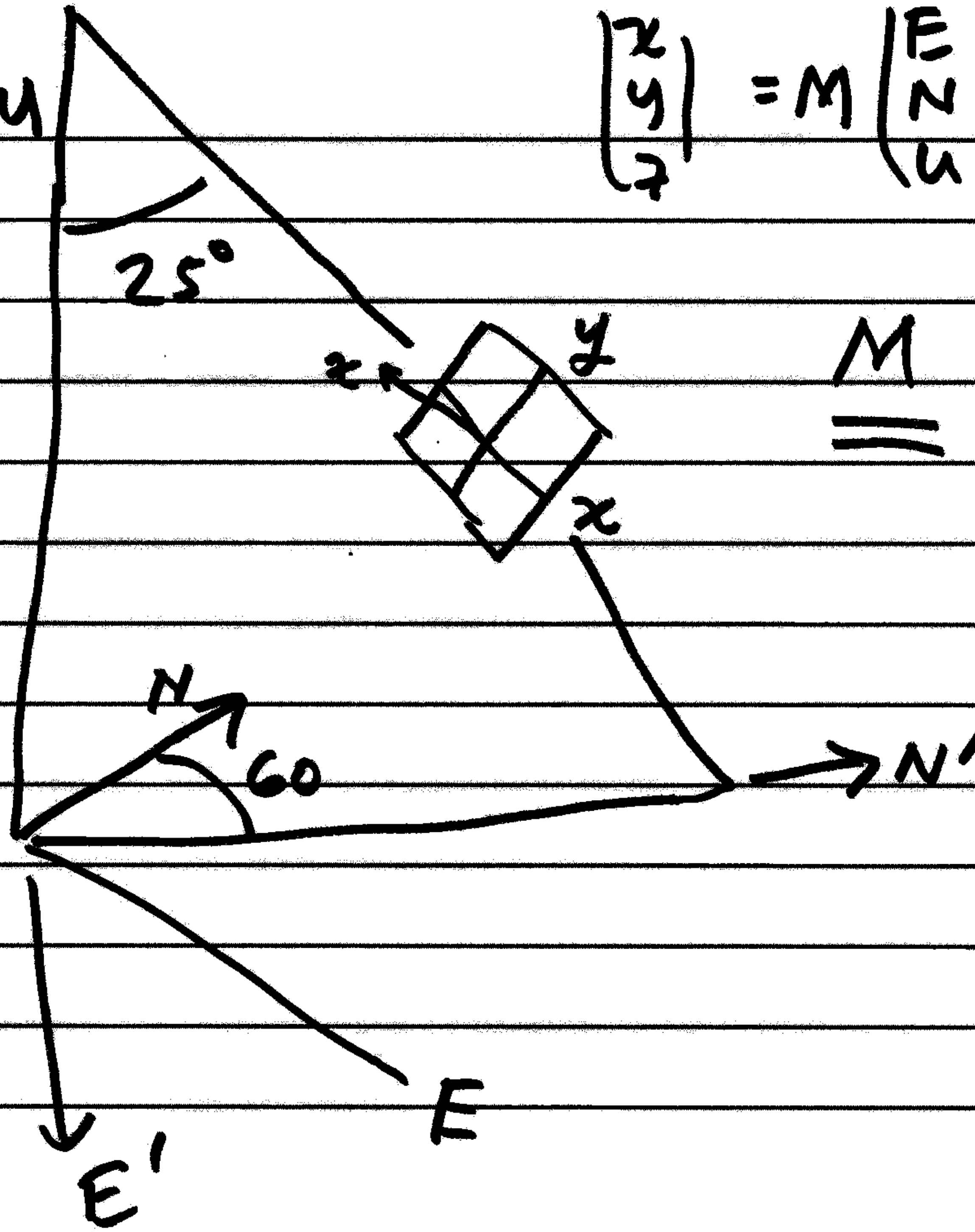
- determinant = +1 or −1



$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} E \\ N \\ u \end{pmatrix} ? \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

$$M = M_k(90°)$$

$$\begin{pmatrix} \cos k & \sin k & 0 \\ -\sin k & \cos k & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

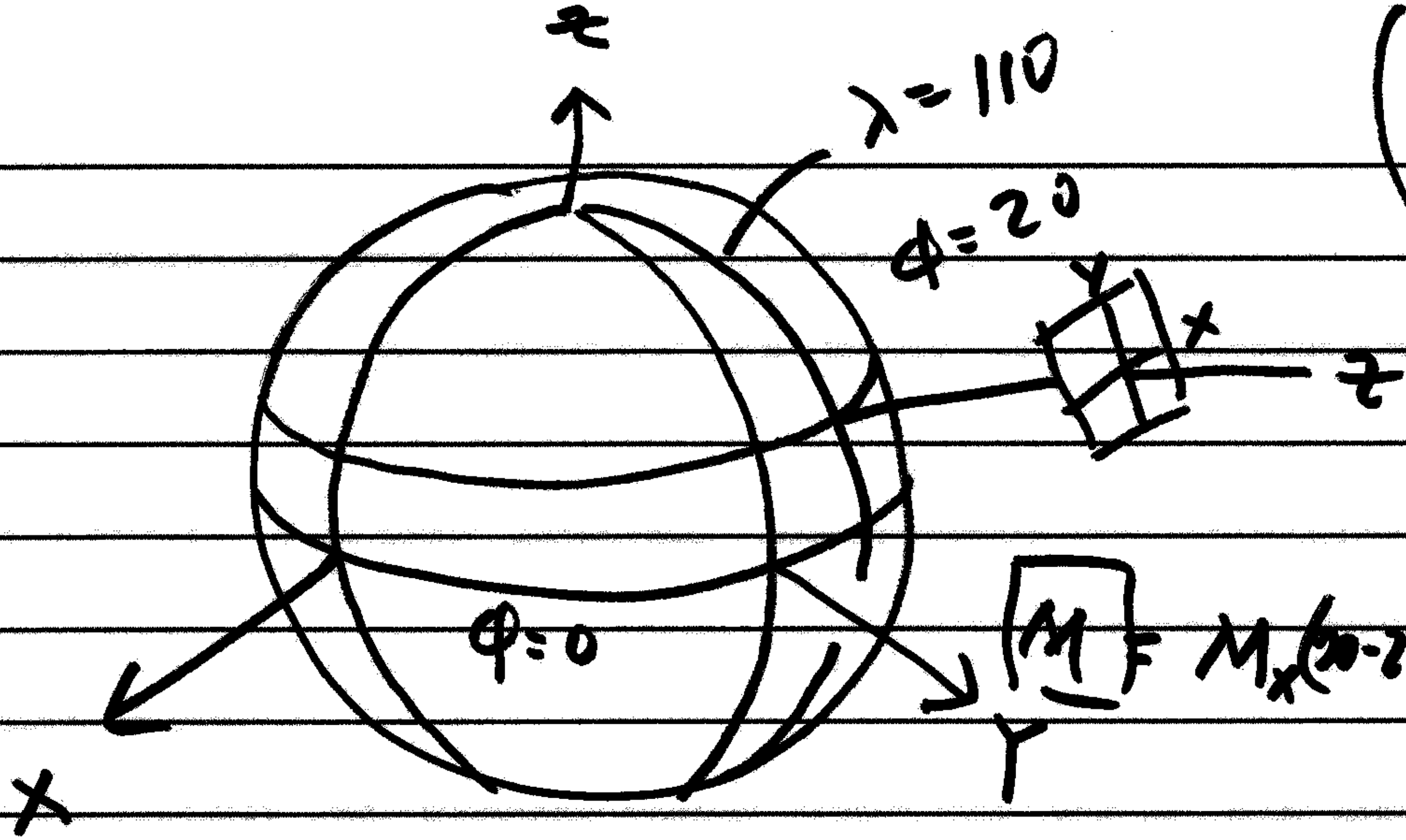


$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} E \\ N \\ u \end{pmatrix}$$

$$M_K M_\phi M_\omega$$

$$\underline{\underline{M}} = M_\omega(25^\circ) M_K(-60^\circ)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}^{65}$$

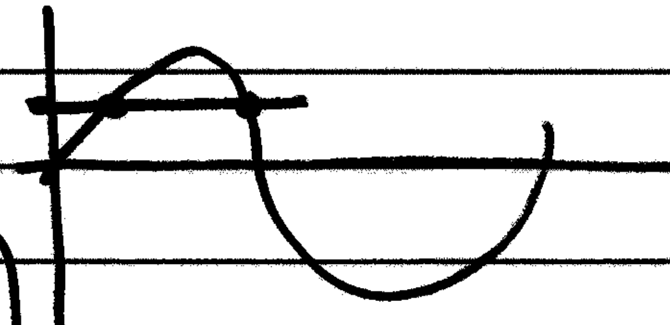


$$\boxed{M} = M_x(90-20)\, M_z(90)\, M_z(110)$$

$M: \underline{\underline{M_k M_\varphi M_\omega}}$ $\quad \omega = ?, \varphi = ?, k = ?$

$$M = \begin{bmatrix} \cos\varphi \cos k & — & — \\ -\cos\varphi \sin k & — & — \\ \sin\varphi & -\sin\omega \cos\varphi & \cos\omega \cos\varphi \end{bmatrix}$$

$$\phi = \sin^{-1}(m_{31})$$



$$\omega = \tan^{-1}\left(\frac{-m_{32}/\cos\phi}{m_{33}/\cos\phi}\right)$$

use 2 argument <u>atan2</u>

$$K = \tan^{-1}\left(\frac{-M_{21}/\cos\phi}{M_{11}/\cos\phi}\right)$$

special case : $\varphi = 90$

$$\begin{bmatrix} 0 & \cos w \sin k + \sin w \cos k & \sin w \sin k - \cos w \cos k \\ 0 & \cos w \cos k - \sin w \sin k & \sin w \cos k + \cos w \sin k \\ 1 & 0 & 0 \end{bmatrix}$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$
$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\begin{bmatrix} 0 & \sin(w+k) & -\cos(w+k) \\ 0 & \cos(w+k) & \sin(w+k) \\ 1 & 0 & 0 \end{bmatrix}$$

any pair $w+k$

<u>gimbal</u> <u>lock</u>

$$q = q_s + q_i i + q_j j + q_k k$$

$$q_s^2 + q_i^2 + q_j^2 + q_k^2 = 1$$

q_s, q_i, q_j, q_k

q_i, q_j, q_k, q_s

quaternion : axis/angle parameters

$\alpha, \beta, \gamma, \theta$

$$\theta = \cos^{-1}\left(\frac{\mathrm{tr}M - 1}{2}\right)$$

$$\mathrm{tr}(M) = M_{11} + M_{22} + M_{33}$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \frac{1}{2\sin\theta}\begin{pmatrix} M_{32} - M_{23} \\ M_{13} - M_{31} \\ M_{21} - M_{12} \end{pmatrix}$$

$\alpha\beta\gamma\theta \Rightarrow q$

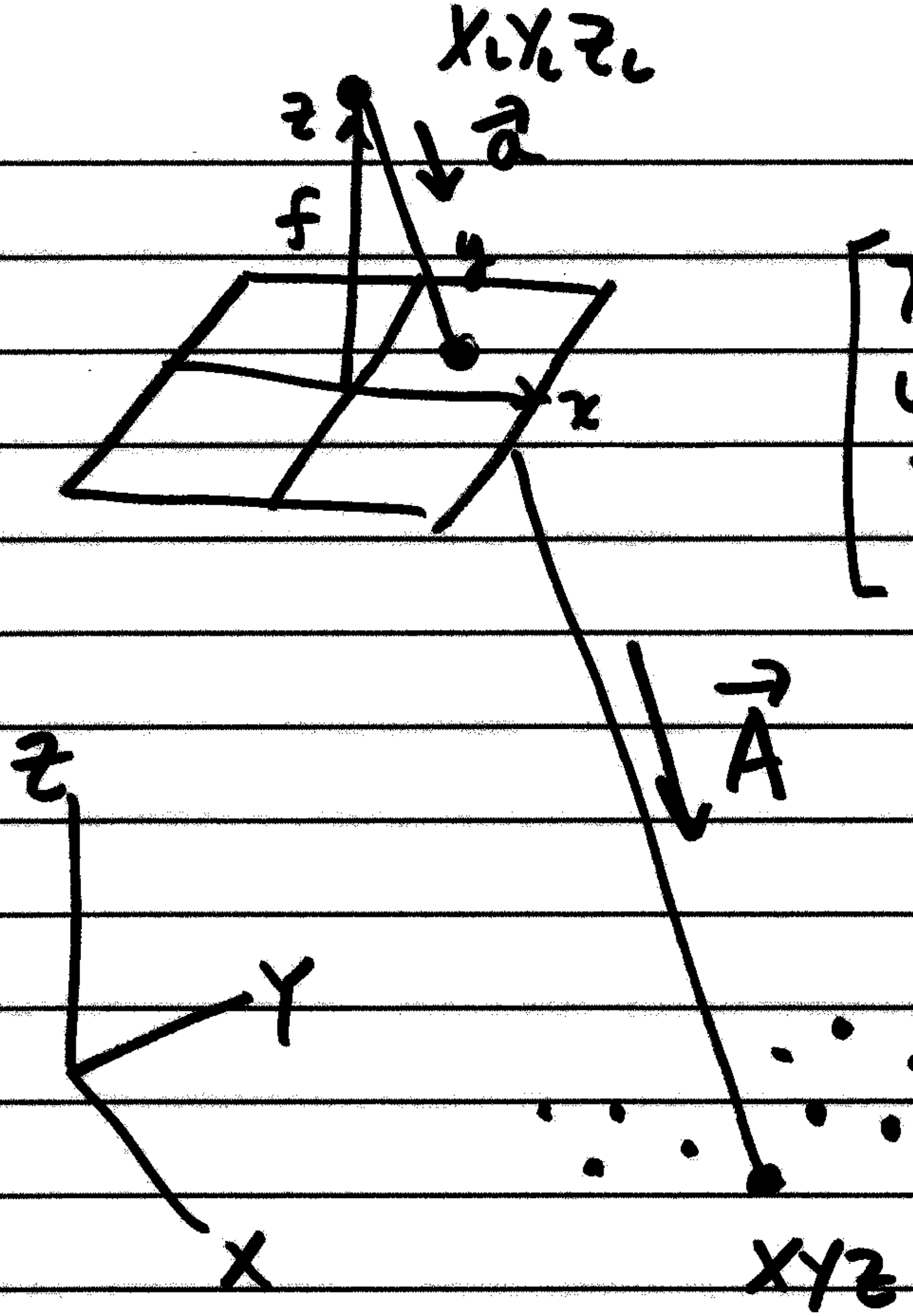
$$q_s = \cos\frac{\theta}{2}$$

$$\begin{bmatrix} q_i \\ q_j \\ q_k \end{bmatrix} = \sin\frac{\theta}{2} \cdot \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

$q \Rightarrow \alpha\beta\gamma\theta$

$$\cos\theta = q_s^2 - \left(q_i^2 + q_j^2 + q_k^2\right)$$

$$\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \frac{1}{\left(q_i^2 + q_j^2 + q_k^2\right)^{1/2}} \begin{bmatrix} q_i \\ q_j \\ q_k \end{bmatrix}$$



$$\vec{a} = \lambda \vec{A}$$

$$\begin{bmatrix} x - x_0 \\ y - y_0 \\ -f \end{bmatrix} = \lambda M \begin{bmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{bmatrix}$$

$$\begin{pmatrix} x - x_0 \\ y - y_0 \\ -f \end{pmatrix} = \lambda \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \begin{pmatrix} X - X_L \\ Y - Y_L \\ Z - Z_L \end{pmatrix}$$

$$\frac{x - x_0}{-f} = \frac{m_{11}(X - X_L) + m_{12}(Y - Y_L) + m_{13}(Z - Z_L)}{m_{31}(X - X_L) + m_{32}(Y - Y_L) + m_{33}(Z - Z_L)}$$

$$x - x_0 = -f \frac{\boxed{\quad\quad}}{\boxed{\quad\quad}}$$

<u>Ground to Image</u>

$$\frac{y - y_0}{-f} = \frac{m_{21}(X - X_L) + m_{22}(Y - Y_L) + m_{23}(Z - Z_L)}{m_{31}(X - X_L) + m_{32}(Y - Y_L) + m_{33}(Z - Z_L)}$$

Collinearity Equations

image $\leftarrow$ Ground

$$\begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \lambda M \begin{pmatrix} X-X_c \\ Y-Y_c \\ Z-Z_c \end{pmatrix}, \quad \frac{1}{\lambda} M^T \begin{pmatrix} x-x_0 \\ y-y_0 \\ -f \end{pmatrix} = \begin{pmatrix} X-X_c \\ Y-Y_c \\ Z-Z_c \end{pmatrix}$$

$$\frac{m_{11}(x-x_0) + m_{21}(y-y_0) + m_{31}(-f)}{m_{13}(x-x_0) + m_{23}(y-y_0) + m_{33}(-f)} (Z-Z_c) + X_c = X$$

$$\frac{m_{12}(x-x_0) + m_{22}(y-y_0) + m_{32}(-f)}{(\cdot)} (Z-Z_c) + Y_c = Y$$

$$\begin{pmatrix} X - X_0 \\ Y - Y_0 \\ -f \end{pmatrix} = \lambda R \, M \underbrace{\begin{pmatrix} (X - X_L)/R \\ (Y - Y_L)/R \\ (Z - Z_L)/R \end{pmatrix}}$$

$$\begin{pmatrix} \\ \\ \\ \end{pmatrix} = \lambda R \, M \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix}$$

Directional Control

Divide first 2 equations by the 3rd one

$$\frac{x - x_0}{-f} = \frac{m_{11} C_x + m_{12} C_y + m_{13} C_z}{m_{31} C_x + m_{32} C_y + m_{33} C_z}$$

$$\frac{y - y_0}{-f} = \frac{m_{21} C_x + m_{22} C_y + m_{23} C_z}{m_{31} C_x + m_{32} C_y + m_{33} C_z}$$