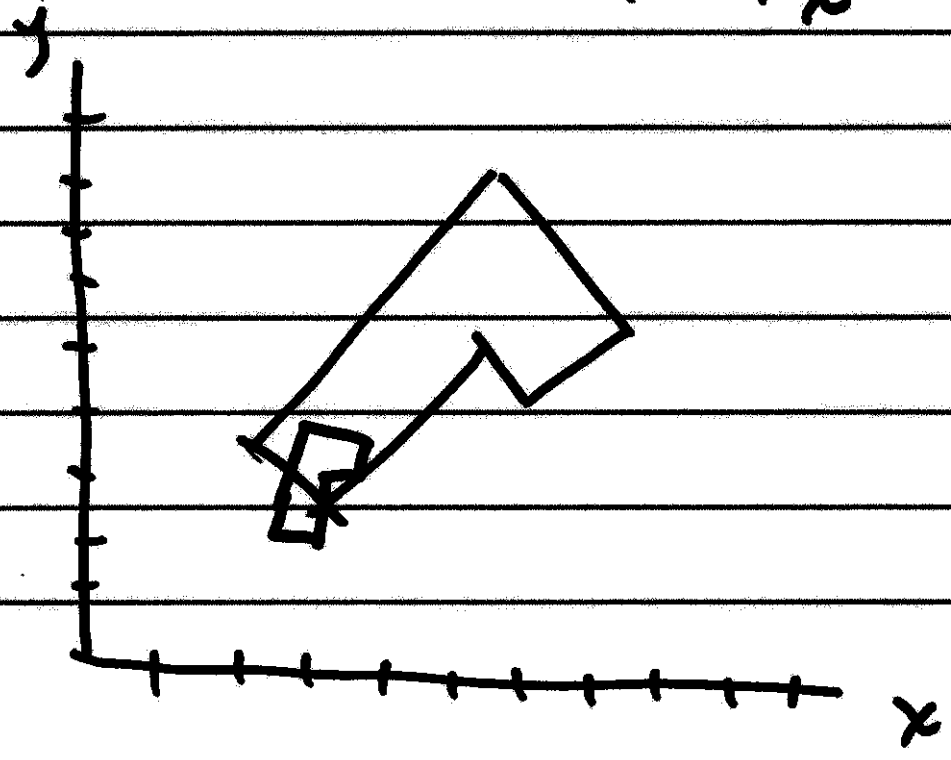
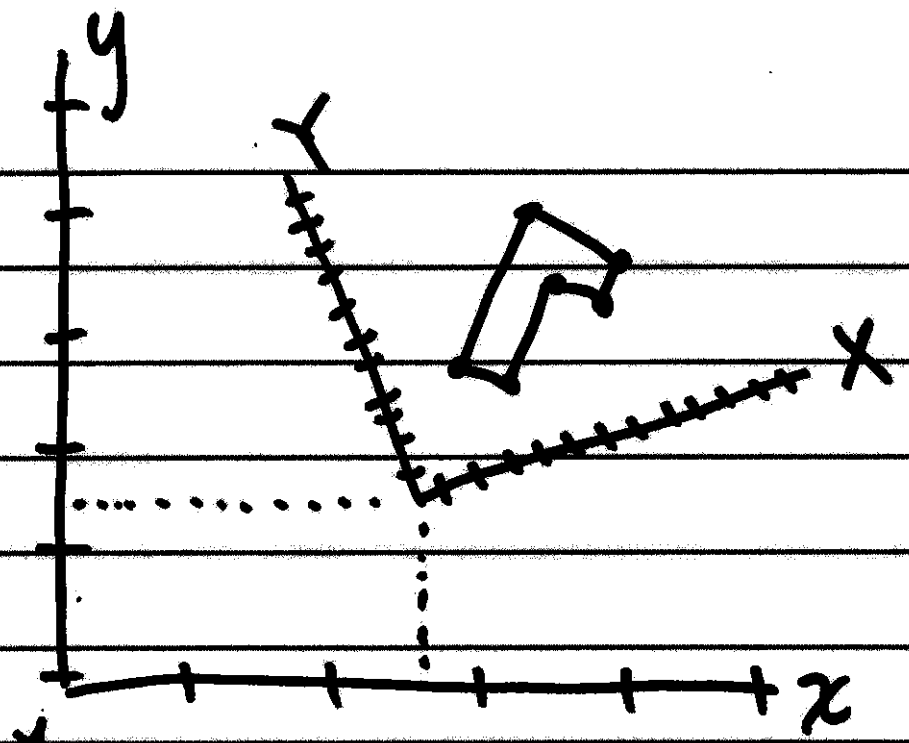


$$\lambda = 0.3$$
$$\theta = -30^\circ$$
$$T_x = 2.5$$
$$T_y = 1.5$$



similarity transformation 2D  
scale, shifts, rotation

$$\begin{pmatrix} x \\ y \end{pmatrix} \leftarrow \begin{pmatrix} X \\ Y \end{pmatrix}^{7-2}$$

$$x = \lambda \cos \theta \cdot X + \lambda \sin \theta Y + T_x$$

$$y = -\lambda \sin \theta X + \lambda \cos \theta Y + T_y$$

$\lambda$ : scale

$\theta$ : rotation

$T_x, T_y$ : shifts

$$\vec{x} = \lambda M \vec{X} + \vec{T}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} T_x \\ T_y \end{pmatrix}$$

= = =

$$x = \overbrace{\lambda \cos \theta}^{=a} X + \overbrace{\lambda \sin \theta}^{=b} Y + \overbrace{T_x}^{=c} \quad z^{-3}$$

$$y = \underbrace{-\lambda \sin \theta}_b X + \underbrace{\lambda \cos \theta}_{a''} Y + \cancel{T_y} = d$$

$$\begin{aligned} x &= aX + bY + c \\ y &= -bX + aY + d \end{aligned}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} X & Y & 1 & 0 \\ Y & -X & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$(x_1, y_1) \quad (X_1, Y_1) \quad x_2 y_2 \quad X_2 Y_2 \quad x_3 y_3 \quad X_3 Y_3 \quad 7-4$$

$$\begin{pmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 & y_1 & 1 & 0 \\ y_1 & -x_1 & 0 & 1 \\ x_2 & y_2 & 1 & 0 \\ y_2 & -x_2 & 0 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

~~$$x = Ax$$~~

$$x = Ap$$

$$p = A^{-1}x$$

$$a = \lambda \cos \theta$$

$$b = \lambda \sin \theta$$

$$a^2 + b^2 = \lambda^2 \cos^2 \theta + \lambda^2 \sin^2 \theta$$

$$\lambda^2 (\cos^2 \theta + \sin^2 \theta)$$

"  
1

$$\underline{\underline{\lambda}} = \sqrt{a^2 + b^2}$$

$$\Theta = \tan^{-1}(b/a)$$

7-5

$$T_x = c$$

$$T_y = d$$

2D similarity transformation

3D Sim. Transf.

$$\underline{x = \lambda M X + T} \quad 7-6$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \lambda \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix}$$

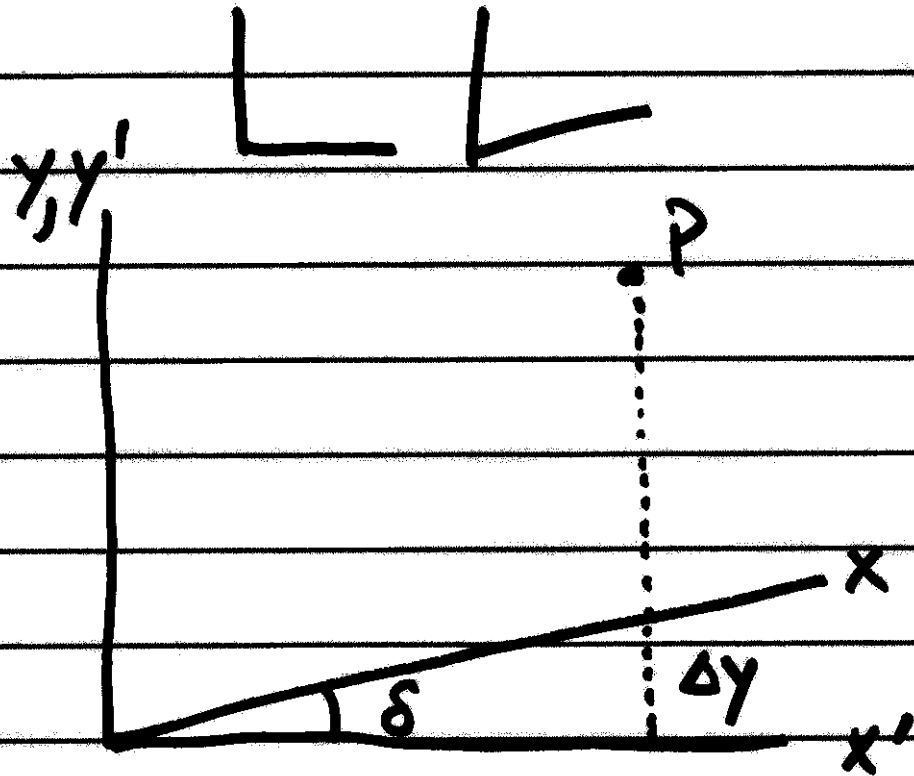
$\lambda$ : scale  
 $\omega, \phi, \kappa$ : rotation  
 $T_x, T_y, T_z$ : shifts

} 7 parameters

# affine transformation 2D

7-7

scale, rotation, shifts, scale, non-orthogonality



$$\left. \begin{aligned} \frac{x'}{x} &= \cos \delta \\ \frac{\Delta y}{x} &= \sin \delta \end{aligned} \right\} = \delta \text{ small}$$

$$x' = x$$

$$y' = y + \Delta y = y + \delta \cdot x$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

↑  
scale

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \delta & 1 \end{pmatrix} \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x''' \\ y''' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \delta & 1 \end{pmatrix} \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

6 parameter transformation

$s_x, s_y, \delta, \theta, t_x, t_y$



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} S_x (\cos\theta + \delta \sin\theta) & S_y \sin\theta \\ S_x (-\sin\theta + \delta \cos\theta) & S_y \cos\theta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

$\begin{matrix} \text{" } a_1 \\ b_1 \end{matrix} \qquad \qquad \qquad \begin{matrix} \text{" } a_2 \\ b_2 \end{matrix} \qquad \qquad \qquad \begin{matrix} \text{" } a_0 \\ b_0 \end{matrix}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_0 \\ b_0 \end{pmatrix} + \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & X & Y & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & X & Y \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{pmatrix}$$

$$S_y = \sqrt{a_2^2 + b_2^2}$$

7-10

$$\Theta = \tan^{-1}(a_2/b_2)$$

$$S_x = \frac{a_1 b_2 - b_1 a_2}{S_y}$$

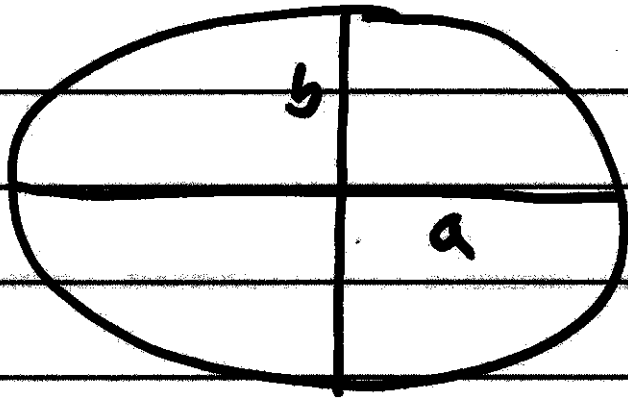
$$\delta = \frac{a_1 - S_x \cos \Theta}{S_x \sin \Theta}$$

$$t_x = a_0$$

$$t_y = b_0$$

reference ellipsoid

7-11



$a, b$

$a, 1/f$

$f = \frac{a-b}{a}$  flattening

WGS 84

$$e^2 = \frac{a^2 - b^2}{a^2}$$

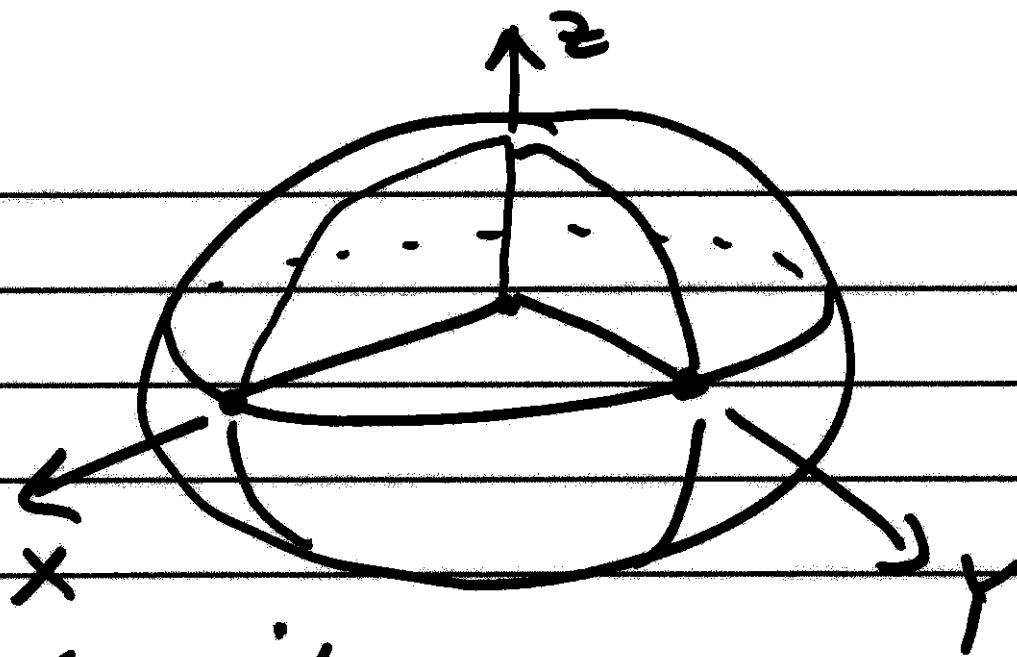
$$e^2 = 2f - f^2$$

$$b^2 = a^2(1 - e^2)$$

$$a = 6378137.0 \text{ m}$$

$$1/f = 298.257223563$$

$$a - b \approx 21 \text{ km}$$



XYZ : Geocentric System

ECEF : earth centered  
earth fixed

ECEF

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ECI

Greenwich  
 $\lambda = 0$

$\phi, \lambda, h$

